MATH 141, FALL 2016, HW10

DUE IN SECTION, NOVEMBER 15

Problem 1. (Apostol p.311 #6) Find all solutions of $y' \sin x + y \cos x = 1$ on the interval $(0, \pi)$. Prove that exactly one of these solutions has a finite limit as $x \to 0$, and another has a finite limit as $x \to \pi$.

Problem 2. (Apostol, p.311 #8) Find all solutions of $y' + y \cot x = 2 \cos x$ on the interval $(0, \pi)$. Prove that exactly one of these is also a solution on $(-\infty, \infty)$.

Problem 3. (Apostol, p.320 #8) A thermometer has been stored in a room whose temperature is 75F. Five minutes after being taken outdoors it reads 65F. After another five minutes, it reads 60F. Compute the outdoor temperature.

Problem 4. (Apostol, p.320 #11) Consider an electric circuit with inductance L, resistance R and an alternating generator which produces a voltage $V(t) = E \sin \omega t$, where E and ω are positive constants. If $I(0) = 0$, prove that the current has the form

$$I(t) = \frac{E}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \alpha) + \frac{E \omega L}{R^2 + \omega^2 L^2} e^{-Rt/L}.$$

Problem 5. (20 pts) Determine the general solution of the following differential equations

(1) $y'' + y = \sin x$
(2) $y'' - 3y' = 2e^{2x} \sin x$
(3) $y'' + 4y = 3x \sin x$
(4) $y'' + y' - 2y = e^x + e^{2x}$.

Problem 6. Let $a < b$ and let $f : [a, b] \to \mathbb{R}$ be a continuously differentiable function satisfying $f(a) = 0$. Prove

$$\int_a^b f(x)^2 dx < (b-a)^2 \int_a^b f'(x)^2 dx.$$

Bonus Problem. Suppose f is a real, continuously differentiable function on $[a, b]$, $f(a) = f(b) = 0$, and

$$\int_a^b f^2(x) dx = 1.$$

Prove that

$$\int_a^b x f(x) f'(x) dx = -\frac{1}{2}$$

and that

$$\int_a^b [f'(x)]^2 dx \cdot \int_a^b x^2 f^2(x) dx > \frac{1}{4}.$$