MATH 141, FALL 2016, HW1

DUE IN SECTION, SEPTEMBER 1

Problem 1. Prove de Morgan's law: Let X be a set and let A and B be subsets. Then

$$(A \cap B)^c = A^c \cup B^c, \qquad (A \cup B)^c = A^c \cap B^c.$$

Problem 2. Let $P(m,n)$ be a statement about $\mathbb{N}^2$. Show that P is true of all of $\mathbb{N}^2$ if the following statements are true:

(1) $P(0,0)$
(2) For all $m \in \mathbb{N}$, $P(0,m)$ and $P(m,0)$
(3) For all $(m,n) \in \mathbb{N}^2$, $P(m,n) \Rightarrow P(S(m),S(n))$.

Problem 3. Prove the trichotomy law for $\mathbb{N}$ from Lecture: for all $(m,n) \in \mathbb{N}^2$ exactly one of $m < n$, $m = n$, $m > n$ is true. Furthermore, if $m < n$ then $S(m) \leq n$.

For the remaining problems you may assume any properties of $\mathbb{N}$ stated in Lecture, and need not use the Polish notation.

Problem 4. Prove by induction $\sum_{i=1}^{n}(2i-1) = n^2$.

Problem 5. (Division algorithm) Let b denote a fixed positive integer. Prove the following statement by induction: for every integer n there exist unique integers q and r such that

$$n = qb + r, \qquad 0 \leq r < b.$$

(The **int** class of the C programming language defines $\frac{n}{b} = q$.)

Bonus Problem. The Euclidean plane is divided into regions by drawing a finite number of straight lines. Prove that it is possible to color each region red or blue in such a way that no pair of adjacent regions have the same color.