MATH 141, FALL 2016 PRACTICE MIDTERM 1

SEPTEMBER 28

Solve 4 of 6 problems. You may quote any result stated during lecture, so long as you represent the result accurately.

Problem 1. Prove by induction

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

Problem 2. Given a function f on $\mathbb{N}$, we say $\lim_{n\to\infty} f(n) = A$ if, for every $\epsilon > 0$ there exists $N > 0$ such that $n > N$ implies $|f(n) - A| < \epsilon$. Evaluate

$$\lim_{n\to\infty} n^{-3/2} \sum_{k=1}^{n} \sqrt{k}.$$

Problem 3. Prove that no order can be defined in the complex field that turns it into an ordered field.

Problem 4. A complex number z is said to be *algebraic* if there are integers $a_0, a_1, ..., a_n$, not all 0, such that

$$a_0 + a_1 z + \cdots + a_n z^n = 0.$$

Prove that the set of algebraic numbers is countable. Is every real number algebraic?

Problem 5. Let $A_a^b(f)$ denote the average of integrable function f on an interval $[a, b]$. Suppose that f is integrable on every sub-interval of $[a, b]$. If $a < c < b$, prove that there is a number t satisfying $0 < t < 1$ such that $A_a^b(f) = tA_a^c(f) + (1-t)A_c^b(f)$. Thus A_a^b is a weighted average of A_a^c and A_c^b.

Problem 6. Give the proof of the following theorem from lecture. Let f be a continuous function, and suppose $f(c) > 0$. Then there is a neighborhood $N(c, \delta)$ such that $f(x) > 0$ for all $x \in N(c, \delta)$.