MATH 141, FALL 2016 PRACTICE FINAL

DECEMBER 15

Solve 6 of 8 problems. You may quote results stated during lecture, so long as you represent the result accurately.

Problem 1.

a. (2 points) State carefully the definition of the supremum of a set which is bounded above.

b. (3 points) Prove that a sequence which is increasing and bounded above converges to it's supremum.

Problem 2.

a. (2 points) State the Intermediate Value Theorem.

b. (3 points) Let $f : [0,1] \to [0,1]$ be continuous. Prove that there is $c \in [0,1]$ such that $f(c) = c$.

Problem 3.

a. (2 points) Give the definition of an uniformly continuous function on a closed interval $[a, b]$.

b. (3 points) Give the proof from lecture that a continuous function on a closed interval $[a, b]$ is bounded.

Problem 4. Evaluate the following limits.

a. (3 points)

$$\lim_{n\to\infty}\left(\frac{2n!}{n!\cdot n^n}\right)^{\frac{1}{n}}.$$

b. (2 points)

$$\lim_{x\to 0}\frac{\log(1+x)-x}{1-\cos x}.$$

Problem 5. Determine whether each series converges. If the series converges, determine whether it converges absolutely.

a. (3 points)

$$\sum_{n=1}^{\infty}(-1)^n\left(\frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots 2n}\right).$$

b. (2 points)

$$\sum_{n=1}^{\infty}\frac{1-n\sin(1/n)}{n}.$$

Problem 6.

a. (2 points) Determine the degree 5 Taylor polynomial of the function $f(x) = \sin x \cos x^2$.

b. (3 points) Determine the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{n! z^n}{n^n}$.

Problem 7.

a. (2 points) Give the definition of a sequence of functions f_n which converges uniformly to a function f on the interval $[a, b]$.

b. (3 points) Prove that the sequence of partial sums of the series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges uniformly on every closed interval $[a, b] \subset \mathbb{R}$.

Problem 8.

a. (3 points) Let $f(x) = (x - 1/2)^2$ on $[0, 1]$. Calculate the Fourier coefficients $\hat{f}(n)$ in the Fourier series of f.

b. (2 points) Prove that the series $\sum_{n=-\infty}^{\infty} \hat{f}(n)e^{2\pi inx}$ converges uniformly to $f(x)$ on $[0, 1]$.