MATH 141, FALL 2016 MIDTERM 1

SEPTEMBER 28

Solve 4 of 6 problems. You may quote results stated during lecture, so long as you represent the result accurately.

Problem 1. Prove by induction

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}.$$

Problem 2. Prove the following statements from the field axioms. For all a, $a \cdot 0 = 0 \cdot a = 0$. Also, 0 has no reciprocal.

Problem 3. Let A be a nonempty set of real numbers which is bounded below. Let $-A$ be the set of all numbers $-x$ where $x \in A$. Prove that

$$\inf A = -\sup(-A).$$

Problem 4. Define $\sin^{-1} : [0,1] \to [0, \frac{\pi}{2}]$ to be the inverse function of $\sin x$. Justify that $\sin^{-1}$ is integrable and calculate

$$\int_0^1 \sin^{-1}(t)dt.$$

Problem 5. State carefully the definition of a function continuous at a point p. Then prove that the function $f(x) = \frac{1}{x}$ is a continuous bijection from $(0, 1)$ to $(1, \infty)$.

Problem 6. State the pigeonhole principle. Using it, prove that among 11 numbers in the range 1 to 100, two differ by at most 9.