Convergence of percolation-decorated triangulations to SLE-decorated LQG

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(with Bernardi, Garban, Gwynne, Miller, Sepulveda, Sheffield, and Sun)

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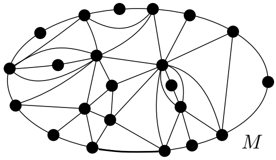
Outline

- Background: discrete and continuum random surfaces
- Part I: Convergence in law of percolation-decorated RPM
- Part II: Conformal embedding of RPM

Joint with:
Bernardi-Sun; Sun; Garban-Sepulveda-Sun; Gwynne-Miller-Sheffield-Sun



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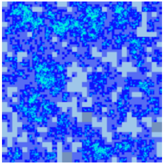

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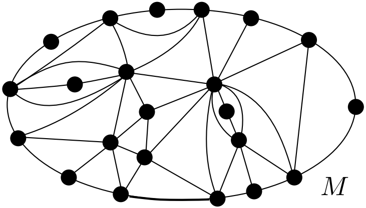

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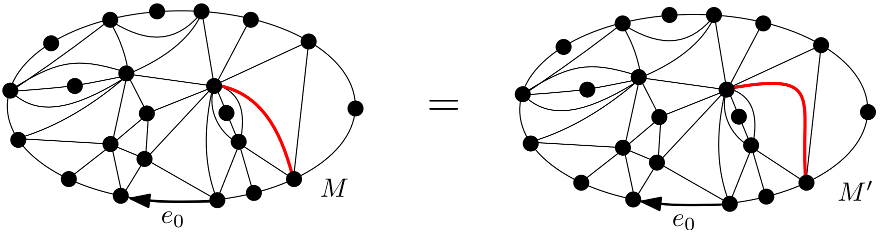

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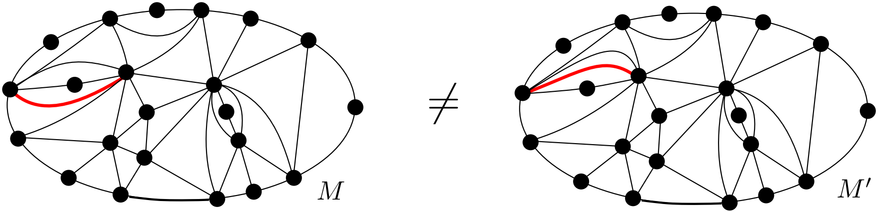

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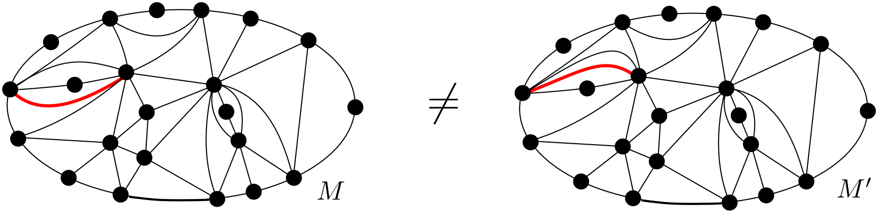

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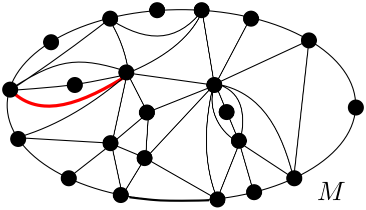
- Given $n, m \in \mathbb{N}$ let M be a **uniformly** chosen triangulation with n vertices and m boundary vertices.



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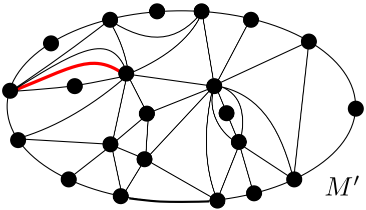

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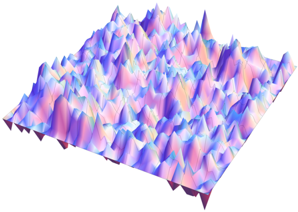

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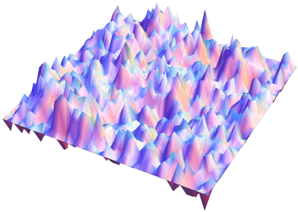

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- The area measure is non-atomic and has full support, but is singular with respect to Lebesgue measure.

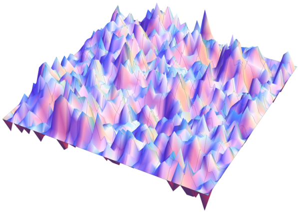

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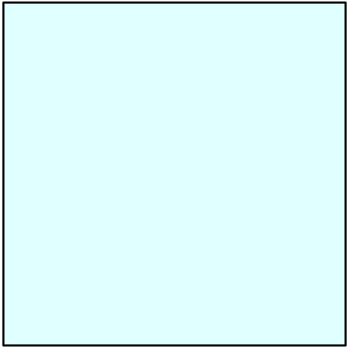
Illustration of LQG area measure



LQG defines a random area measure $e^{\gamma h}\,dx\,dy$ in the square.

Fix $\delta > 0$. Divide squares of LQG area above δ.

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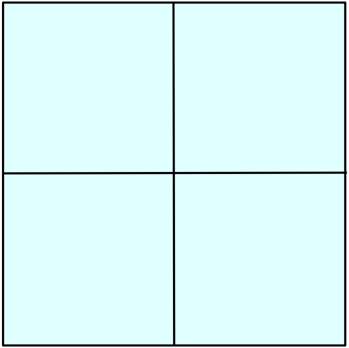

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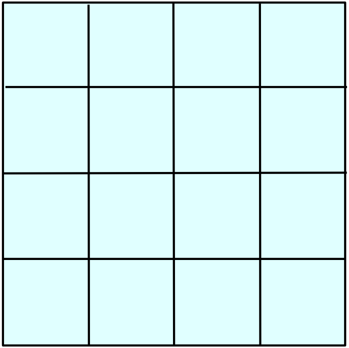

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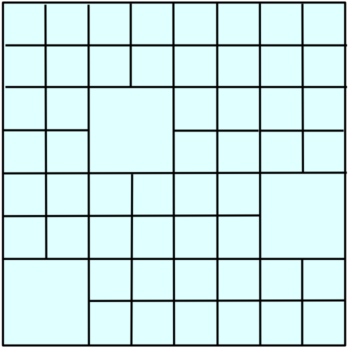

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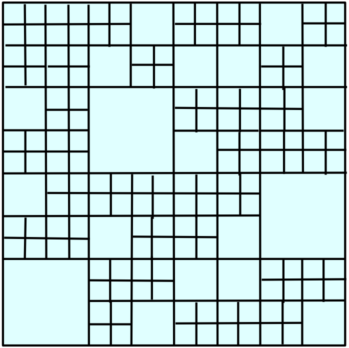

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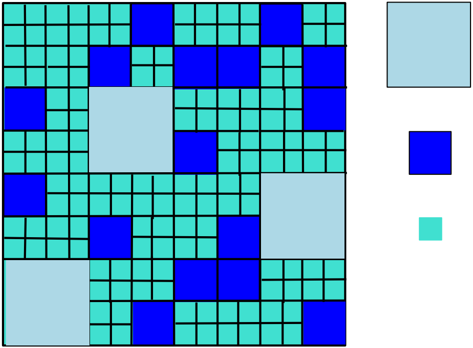

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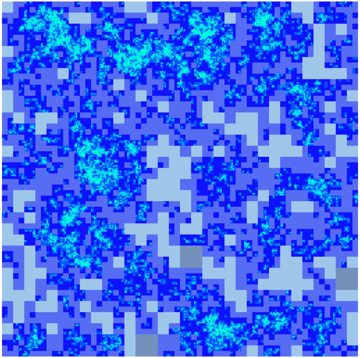

Area measure of random surface $e^{\gamma h} dx\, dy$, $\gamma = 1.5$, by J. Miller

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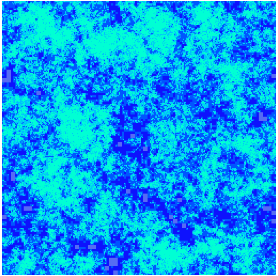

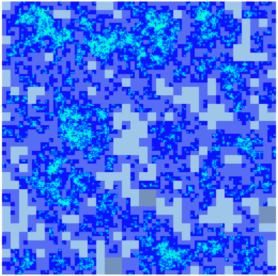

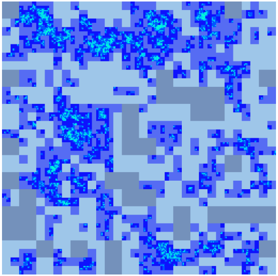

$\gamma = 1$ $\gamma = 1.5$ $\gamma = 1.75$

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Topologies for convergence of RPM:

- Metric space structure (Gromov-Hausdorff topology)
  - Le Gall'13, Miermont'13, and others
- Conformal structure (weak topology for measures on $\mathbb{C}$)
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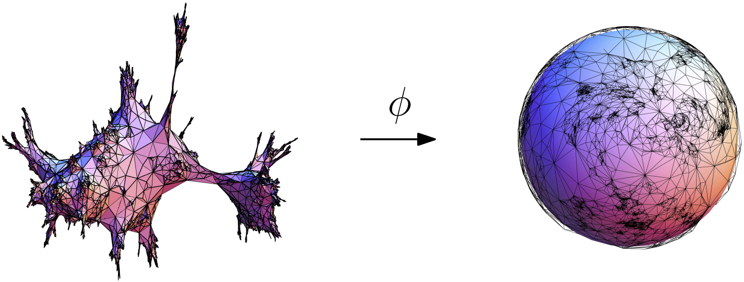
Conjecture: Conformally embedded RPM $\Rightarrow$ LQG

- Let M be a uniformly chosen RPM, and let $\phi : V(M) \to \mathbb{S}^2$ be a discrete conformal map.



RPM M embedded RPM

Figure by Nicolas Curien.

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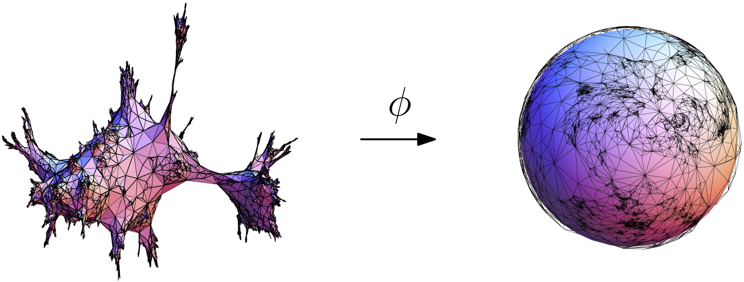

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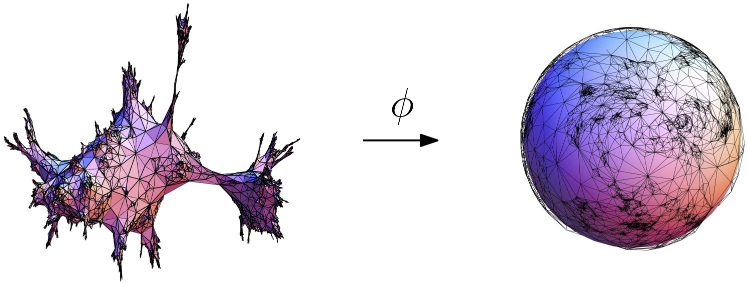
- We get an area measure $\widehat{\mu}_\phi$ on $\mathbb{S}^2$ by considering (renormalized) counting measure induced by $V(M)$.
- $\widehat{\mu}_\phi$ is **conjectured** to converge in law to $\sqrt{8/3}$-LQG area measure μ.



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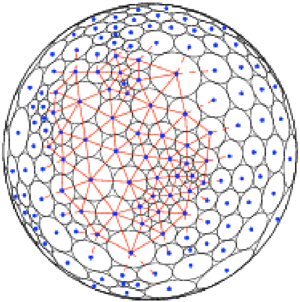
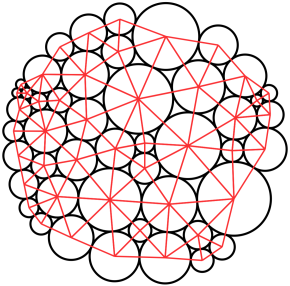
Discrete conformal embeddings

- Circle packing
- Riemann uniformization
- Tutte embedding (harmonic)
- Cardy embedding



circle packing (sphere topology)



circle packing (disk topology)

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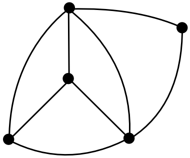

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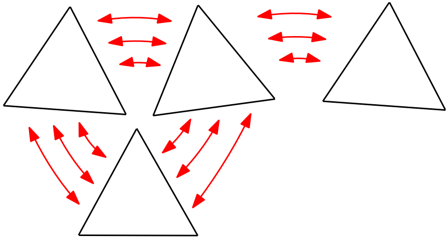

Riemannian manifold

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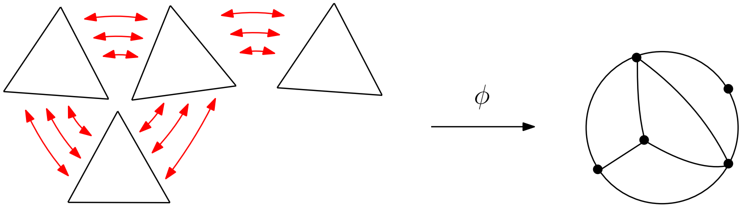
Uniformization theorem: For any simply connected Riemann surface M there is a conformal map ϕ from M to either $\mathbb{D}$, $\mathbb{C}$ or $\mathbb{S}^2$.



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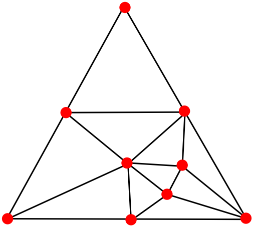

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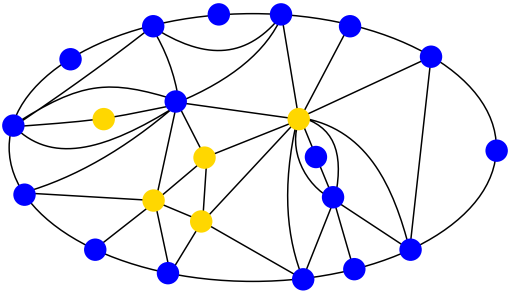

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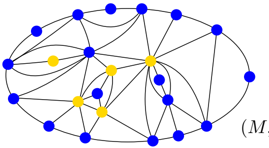
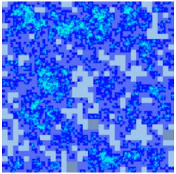
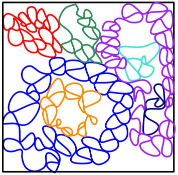
Discrete and continuum decorated surfaces

We **decorate** the surfaces with **percolation** and **CLE$_6$**, respectively:



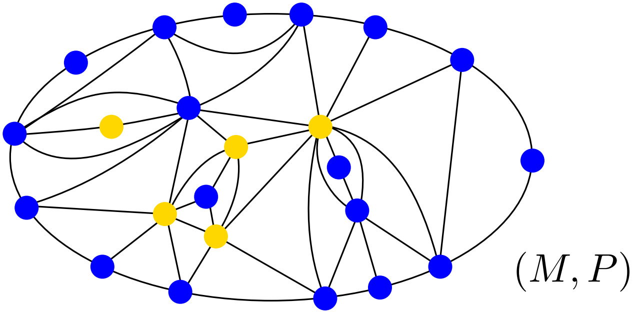




percolation P on RPM M

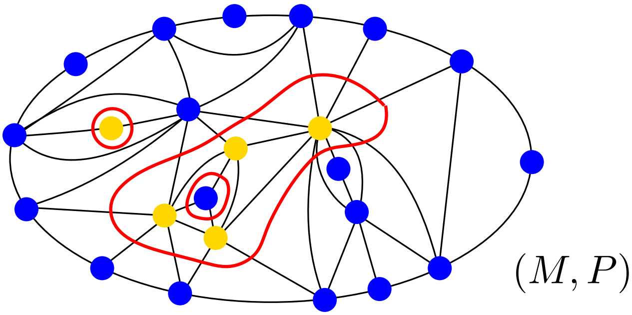
CLE$_6$ Γ on $\sqrt{8/3}$-LQG h

Percolation on RPM



- Consider a uniform triangulation M of the disk.
- Critical percolation probability $p_c^{\text{site}} = 1/2$ (Angel'03).
- We get a percolation P by coloring the inner vertices uniformly and independently blue or yellow, and coloring the boundary vertices blue.

Percolation on RPM



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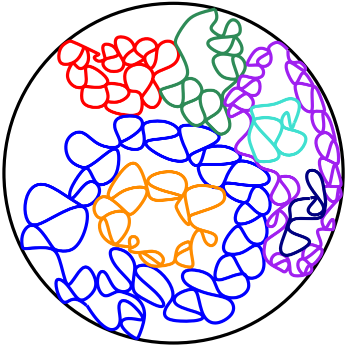

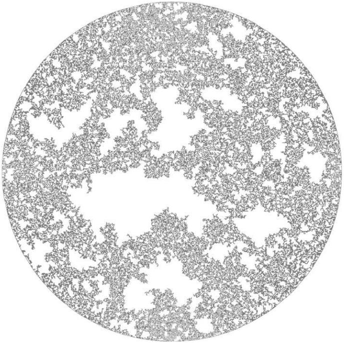

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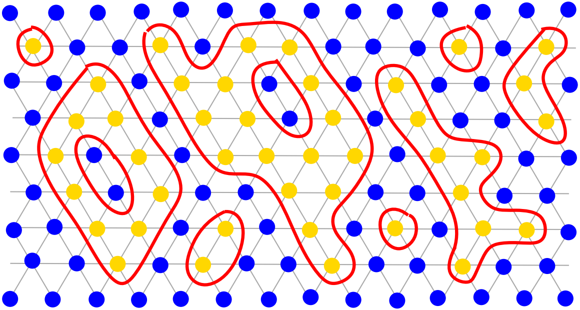

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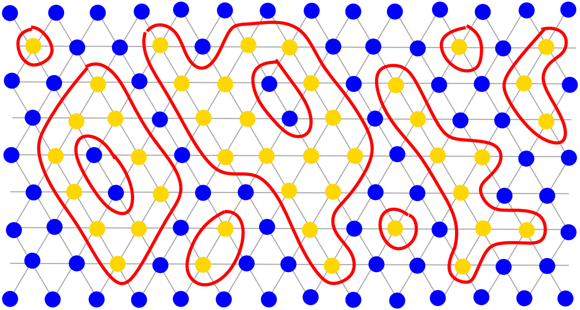

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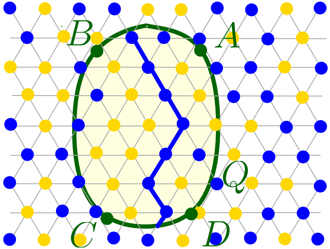

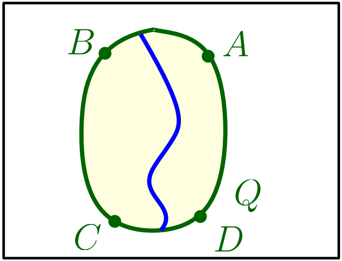

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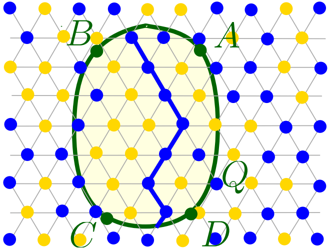
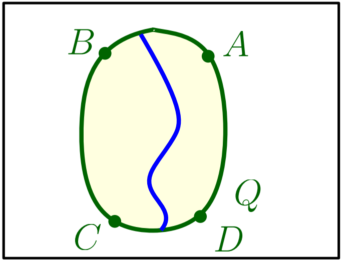
  - η is a space-filling **Schramm-Loewner evolution** SLE_6

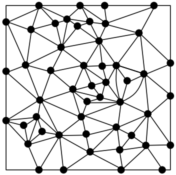




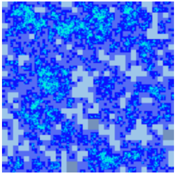
Conjectured relation between **decorated** RPM and LQG

Physics conjectures:

(a) $M \Rightarrow h$ as embedded surfaces



$\Rightarrow$



embedded RPM M $\qquad$ $\sqrt{8/3}$-LQG h

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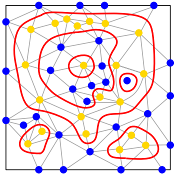
(b) $(M, P) \Rightarrow (h, \Gamma)$ as embedded decorated surfaces



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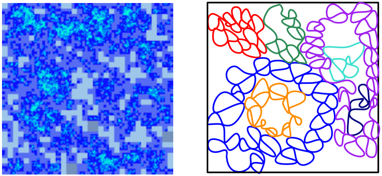


embedded (M, P)

$\sqrt{8/3}$-LQG h and CLE_6 Γ

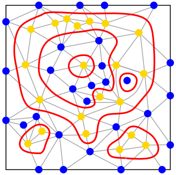
Conjectured relation between **decorated** RPM and LQG

Physics conjectures:

(a) $M \Rightarrow h$ as embedded surfaces

(b) $(M, P) \Rightarrow (h, \Gamma)$ as embedded decorated surfaces

(c) More generally: other decorations give γ-LQG and CLE_κ



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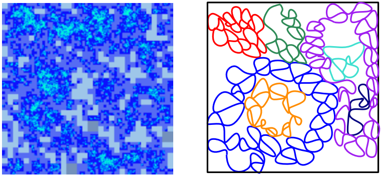

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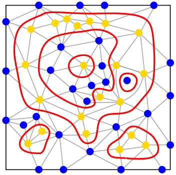

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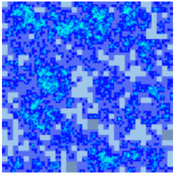
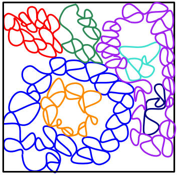




embedded (M, P)

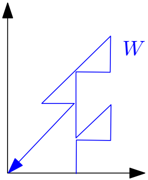
$\sqrt{8/3}$-LQG h and CLE_6 Γ

Peanosphere convergence

map+percolation (M, P) $\overset{\text{bijection}}{\Leftrightarrow}$ walk $(W_k)_{k\in[n]}$

$\Downarrow$ convergence

LQG+CLE (h, Γ) $\overset{\text{measurability}}{\Leftrightarrow}$ Brownian excursion $(Z_t)_{t\in[0,1]}$



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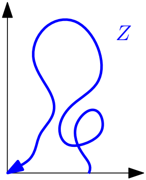


Peanosphere convergence

$$\begin{array}{ccc} \text{map+percolation} & \overset{\text{bijection}}{\Leftrightarrow} & \text{walk } (W_k)_{k\in[n]} \\ (M,P) & & \Downarrow \text{ convergence} \\ \text{LQG+CLE} & \overset{\text{measurability}}{\Leftrightarrow} & \text{Brownian excursion} \\ (h,\Gamma) & & (Z_t)_{t\in[0,1]} \end{array}$$

The result that $W \Rightarrow Z$ (after rescaling) means the following.

Proposition 1

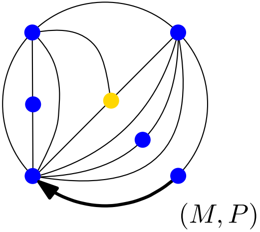
(M, P) *converges to* (h, Γ) *in the* **peanosphere topology** *as introduced by Duplantier-Miller-Sheffield'14.*

Peanosphere encoding of discrete surface

Bernardi'07, Bernardi-H.-Sun'17: Bijection between

(1) site-percolated rooted triangulation (M, P) of disk with $n+1$ edges.

(2) cone excursion W length n, steps $a = (1,0)$, $b = (0,1)$, $c = (-1,-1)$



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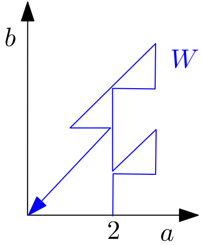

$babcbbabccacc$

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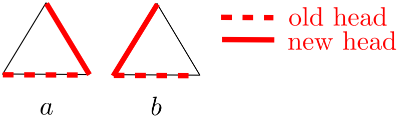

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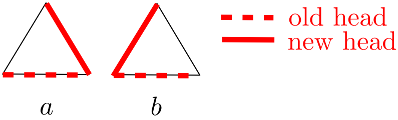

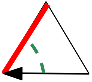

b

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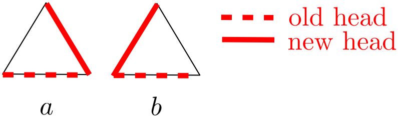

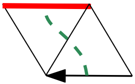

$$ba$$

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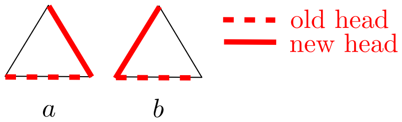

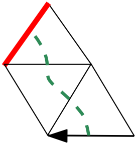

bab

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Peanosphere encoding of discrete surface

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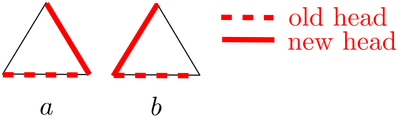

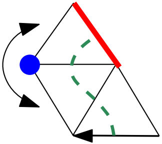

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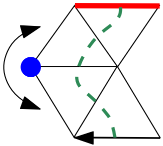

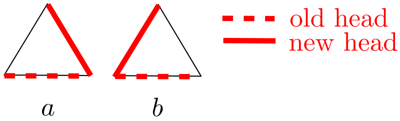

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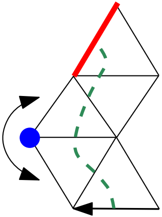

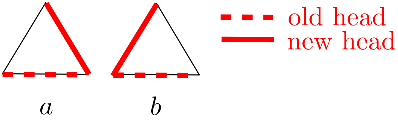

$$babcbb$$

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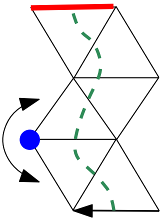

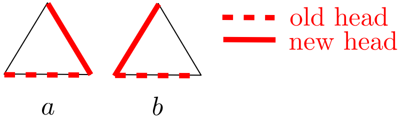

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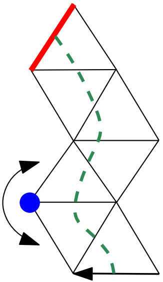

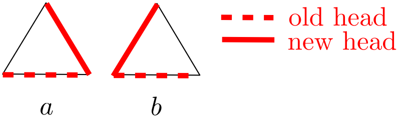

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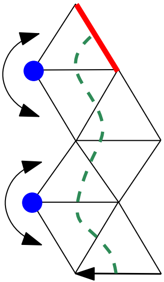

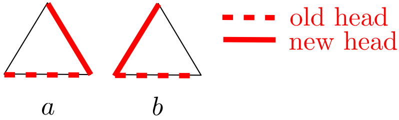

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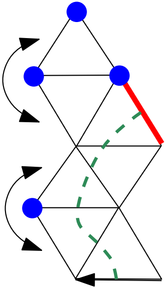

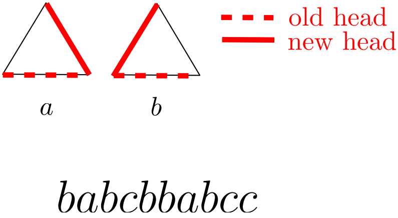

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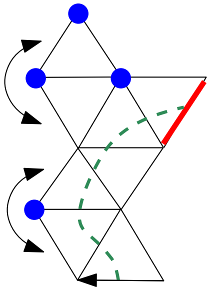

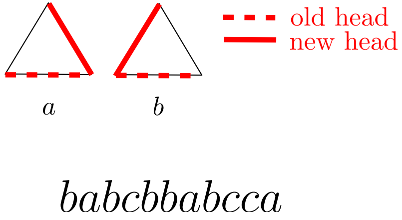

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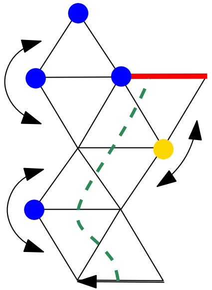

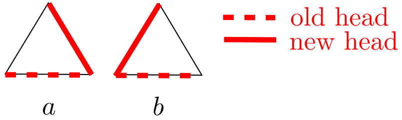

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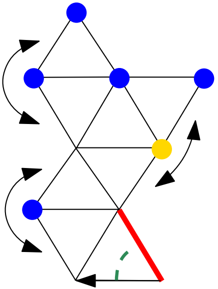

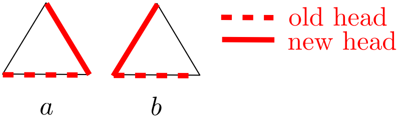

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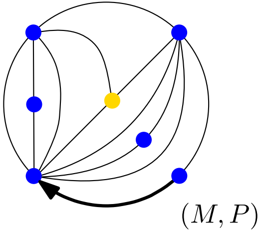

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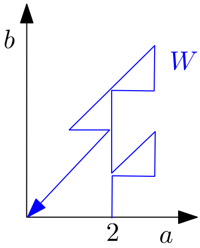

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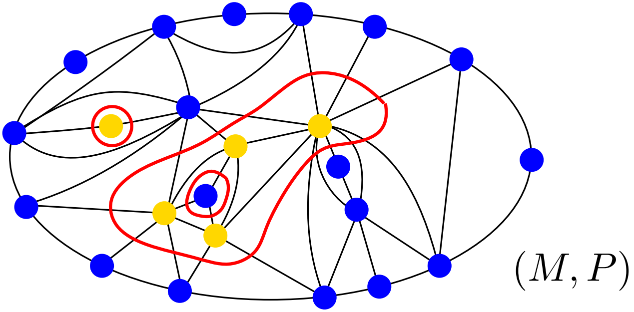
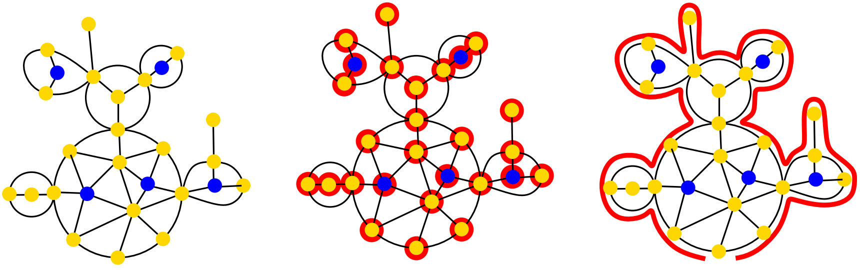
Loop areas, loop lengths, and pivotal measure: discrete



Loop areas, loop lengths, and pivotal measure: discrete

- Let $\mathfrak{a}(C)$ denote the **area** of the cluster C.
- Let $\mathfrak{l}(C)$ denote the **boundary length** of the cluster C.
- Let $C_1, \ldots, C_k$ denote the k clusters with longest boundary.



C $\mathfrak{a}(C)$ $\mathfrak{l}(C)$

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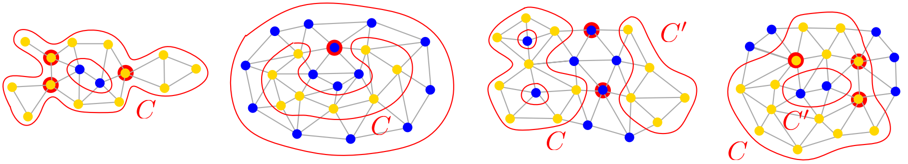
- Let $\mathfrak{l}(C)$ denote the **boundary length** of the cluster C.
- Let $C_1, \ldots, C_k$ denote the k clusters with longest boundary.
- **Pivotal point**: vertex with the property that changing its color makes clusters merge or split
- Let $\mathfrak{p}_1(C)$ and $\mathfrak{p}_2(C)$ denote counting measure on the pivotal points.
- Let $\mathfrak{p}_3(C, C')$ and $\mathfrak{p}_4(C, C')$ denote counting measure on the pivotal points between C and C'.



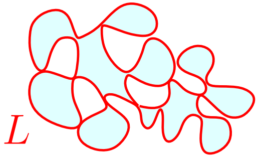
(1) $\mathfrak{p}_1(C)$ (2) $\mathfrak{p}_2(C)$ (3) $\mathfrak{p}_3(C, C')$ (4) $\mathfrak{p}_4(C, C')$

Loop areas, loop lengths, and pivotal measure: continuum

- Consider a $\sqrt{8/3}$-LQG surface, and decorate it with an independent CLE_6 Γ.

Loop areas, loop lengths, and pivotal measure: continuum

- Consider a $\sqrt{8/3}$-LQG surface, and decorate it with an independent CLE_6 Γ.
- Let $a(L)$ denote the **LQG area** enclosed by the CLE loop L.



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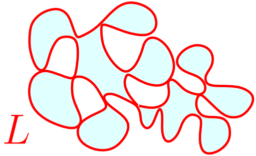

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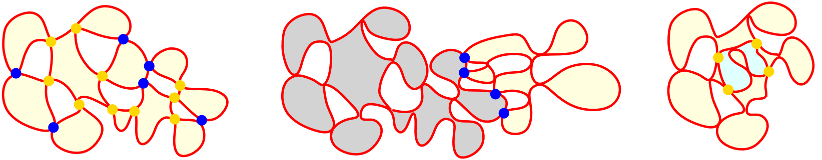
- Let $\ell(L)$ denote the **LQG length** of the CLE loop L.
- Let $p_1(L)$ and $p_2(L)$ denote the **LQG pivotal measure** of L.
- Let $p_3(L, L')$ and $p_4(L, L')$ denote the **LQG pivotal measure** between L and L'.



(1) $p_1(L)$ (2) $p_2(L)$ (3) $p_3(L, L')$ (4) $p_4(L, L')$

Convergence of loops and pivotal measure

- $C_1, \dots, C_k$ are the k clusters of the triangulation with longest boundary.
- $\mathfrak{a}$, $\mathfrak{l}$, and $\mathfrak{p}_m$ denote loop area, loop length, and pivotal measure, respectively.
- Similar continuum notation; $L_1, \dots, L_k$ denote the k longest loops.

Theorem 1 (Bernardi-H.-Sun'17)

Consider a percolation decorated triangulation (M, P) with disk topology. For any $k \in \mathbb{N}$ the following quantities

$$\mathfrak{a}(C_j),\ \ \mathfrak{l}(C_j),\ \ \mathfrak{p}_1(C_j),\ \ \mathfrak{p}_2(C_j),\ \ \mathfrak{p}_3(C_i, C_j),\ \ \mathfrak{p}_4(C_i, C_j), \qquad i, j \in \{1, \dots, k\}$$

converge jointly in law to the associated continuum quantities

$$a(L_j),\ \ \ell(L_j),\ \ p_1(L_j),\ \ p_2(L_j),\ \ p_3(L_i, L_j),\ \ p_4(L_i, L_j), \qquad i, j \in \{1, \dots, k\}.$$

Proof idea

Recall that $(M, P) \Rightarrow (h, \Gamma)$ in the **peanosphere topology**:

$$\begin{array}{ccc} \text{map+percolation} & \overset{\text{bijection}}{\Leftrightarrow} & \text{walk } (W_k)_{k\in[n]} \\ (M,P) & & \Downarrow \ \text{convergence} \\ \text{LQG+CLE} & \overset{\text{measurability}}{\Leftrightarrow} & \text{Brownian excursion} \\ (h,\Gamma) & & (Z_t)_{t\in[0,1]} \end{array}$$

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(M,P) & & \Big\Downarrow \text{convergence of walk and loop quantities encoded by walk} \\
\text{LQG+CLE} & \overset{\text{measurability}}{\Leftrightarrow} & \text{Brownian excursion} \\
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\end{array}
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Outline

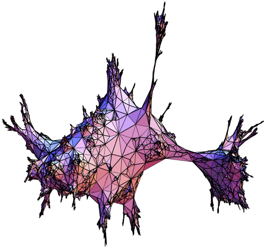
- Background: discrete and continuum random surfaces
- Part I: Convergence in law of percolation-decorated RPM
- Part II: Conformal embedding of RPM

Conformal embedding of planar maps: our goal

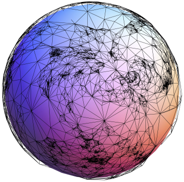
We want a map $\phi : V(M) \to \mathbb{S}^2$ such that

$$\widehat{\mu}_\phi \Rightarrow \mu,$$

where $\widehat{\mu}_\phi$ is the measure on $\mathbb{S}^2$ induced by renormalized counting measure of $V(M)$, and μ is $\sqrt{8/3}$-LQG area measure.



$\xrightarrow{\phi}$



random planar map (RPM) M

embedded RPM

Conformal embedding of planar maps: our goal

We want a map $\phi : V(M) \to \mathbb{T}$ such that

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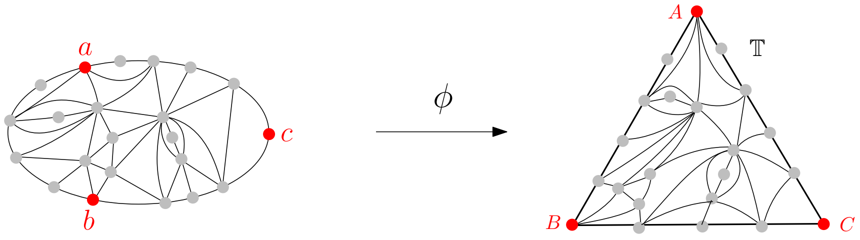

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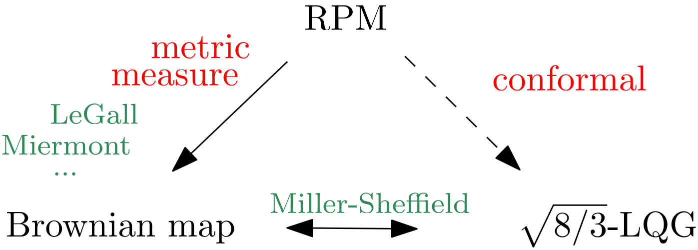
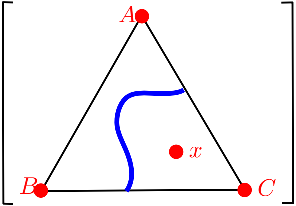


Cardy embedding

- Idea of Cardy embedding: Use properties of **percolation** on the RPM M to determine $\phi : V(M) \to \mathbb{T}$.

$p_1(x) = \mathbb{P}$



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- There is a bijection between $x \in \mathbb{T}$ and triples (p_1, p_2, p_3) of the standard 2-simplex, which is defined in terms of CLE_6 crossing events (Smirnov'01).

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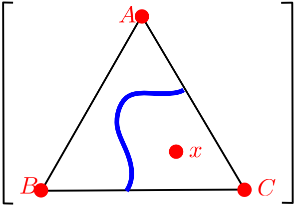

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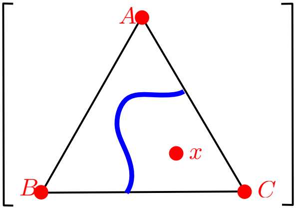

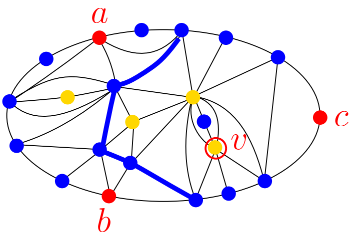

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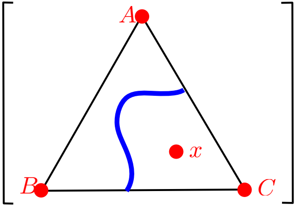
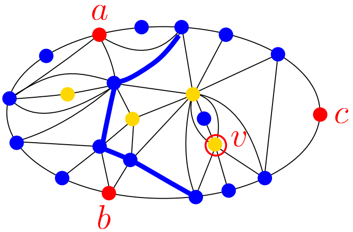
- Given $v \in V(M)$ we can obtain an triple $(\widehat{p}_1, \widehat{p}_2, \widehat{p}_3)$ by considering percolation crossing probabilities on M.
- Let $\phi(v)$ be the point $x \in \mathbb{T}$ associated with the triple $(\widehat{p}_1, \widehat{p}_2, \widehat{p}_3)$ of v.

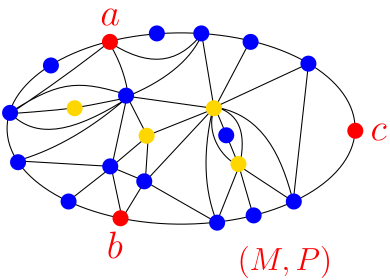
$p_1(x) = \mathbb{P}$

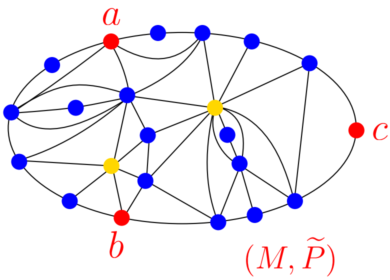




Convergence of the Cardy embedding

- We know that $(M, P) \Rightarrow (h, \Gamma)$ in the peanosphere topology.





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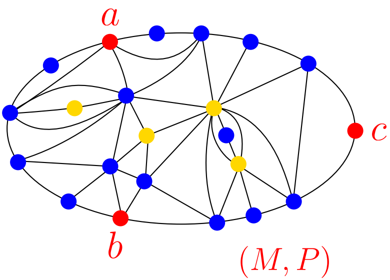

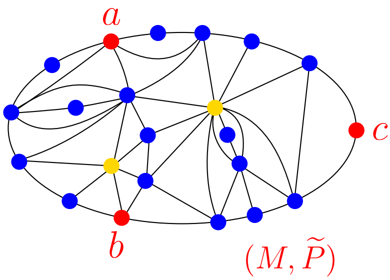

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- We know that $(M, P) \Rightarrow (h, \Gamma)$ in the peanosphere topology.
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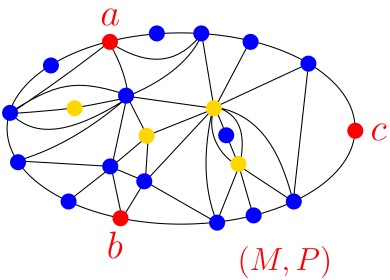

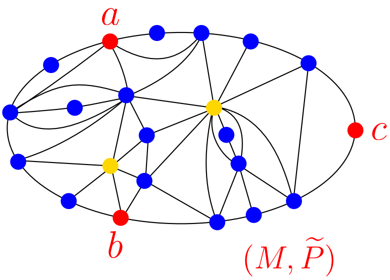

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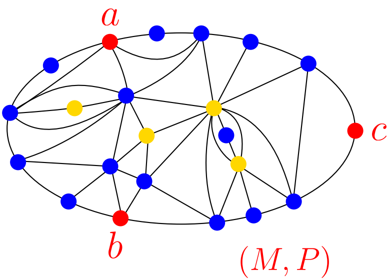

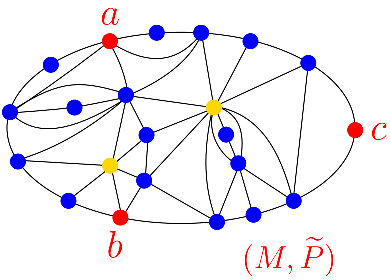

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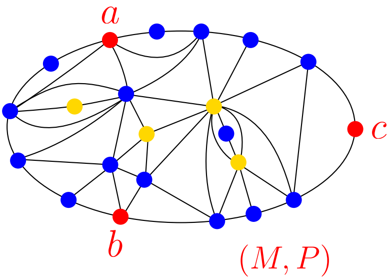
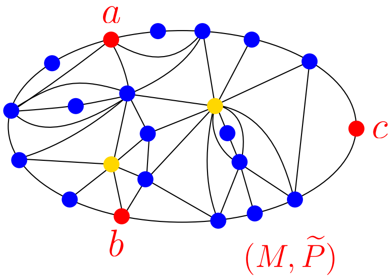
- We know that subsequentially, $\big((M, P), (M, \widetilde{P})\big) \Rightarrow \big((h, \Gamma), (\widetilde{h}, \widetilde{\Gamma})\big)$.
- We believe that
  (a) $h = \widetilde{h}$
  (b) Γ and $\widetilde{\Gamma}$ are independent
- In other words, we believe that $(M, P, \widetilde{P}) \Rightarrow (h, \Gamma, \widetilde{\Gamma})$ for Γ and $\widetilde{\Gamma}$ independent.





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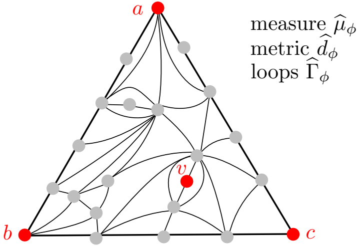
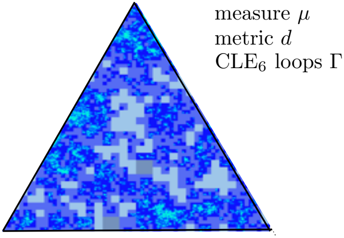


Cardy embedded map (schematic drawing)

$\Longrightarrow$



$\sqrt{8/3}$-LQG/Brownian disk

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Theorem 2 (Gwynne-H.-Miller-Sheffield-Sun'17; assuming (a) & (b))

For a Cardy embedded map with percolation, $(\widehat{\mu}_\phi, \widehat{d}_\phi, \widehat{\Gamma}_\phi) \Rightarrow (\mu, d, \Gamma)$.

Joint convergence in metric and peanosphere topology

- Subsequentially, $\big(M, (M, P), (M, \widetilde{P})\big) \Rightarrow \big(h', (h, \Gamma), (\widetilde{h}, \widetilde{\Gamma})\big)$, where
  - $h', h, \widetilde{h}$ LQG and $\Gamma, \widetilde{\Gamma}$ CLE_6
  - 1st coordinate: metric topology
  - 2nd and 3rd coordinates: peanosphere topology

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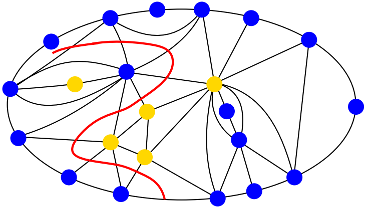
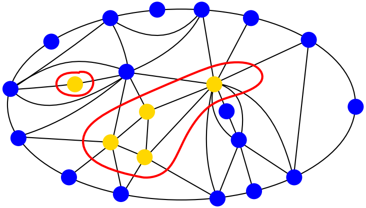
Gwynne-Miller proved joint metric and peanosphere convergence for a map with a **single** percolation interface. We iterate this result.



One percolation interface



All percolation interfaces

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- Combining the above, subsequentially

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Joint convergence in metric and peanosphere topology

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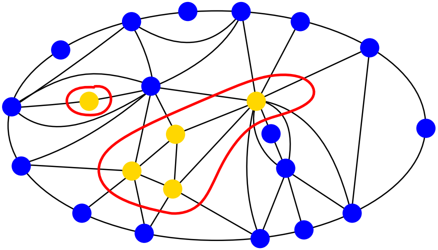
- In particular, $h=\widetilde{h}$, so (a) is established.

Independent percolations give independent CLEs

- Subsequentially, $(M, P, \widetilde{P}) \Rightarrow (h, \Gamma, \widetilde{\Gamma})$ for Γ and $\widetilde{\Gamma}$ not necessarily independent.
- We want to prove (b) independence of Γ and $\widetilde{\Gamma}$.



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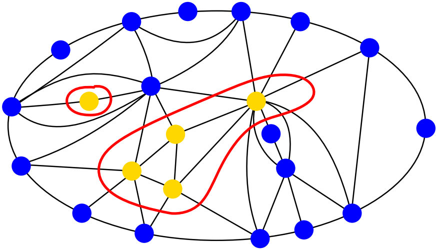

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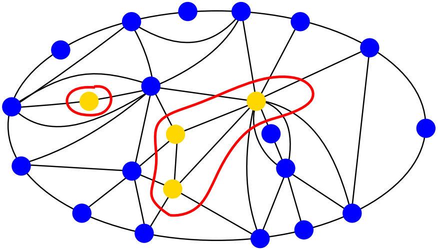

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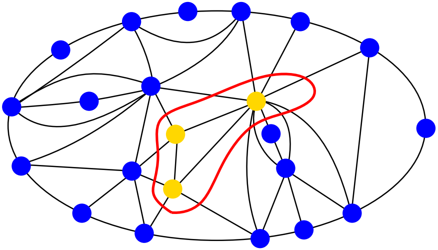

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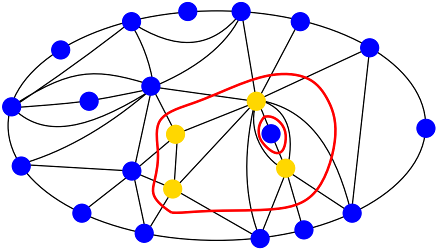

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  (1) $(M, (P_t)_{t\geq 0}) \Rightarrow (h, (\Gamma_t)_{t\geq 0})$, for $(\Gamma_t)_{t\geq 0}$ continuum quantum DP (QDP).
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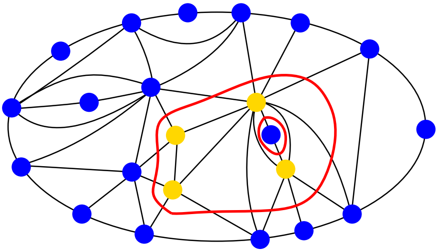

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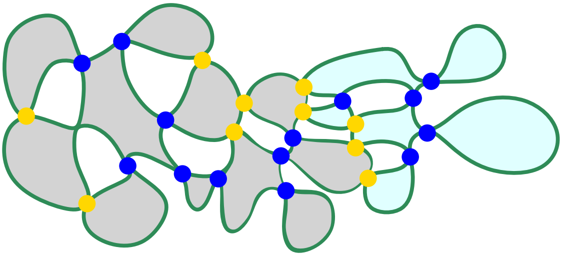

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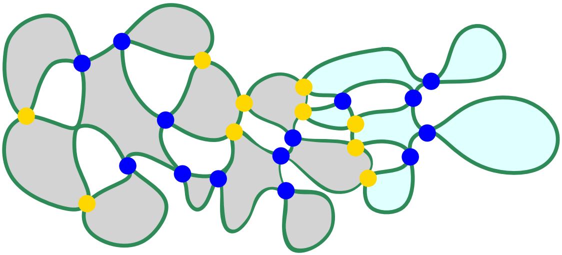

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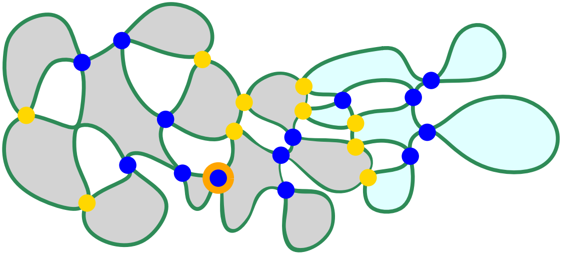

Independent percolations give independent CLEs

- Subsequentially, $(M, P, \widetilde{P}) \Rightarrow (h, \Gamma, \widetilde{\Gamma})$ for Γ and $\widetilde{\Gamma}$ not necessarily independent.
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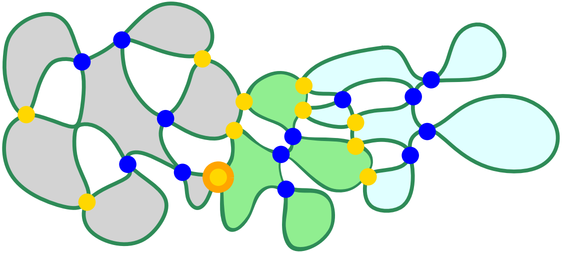

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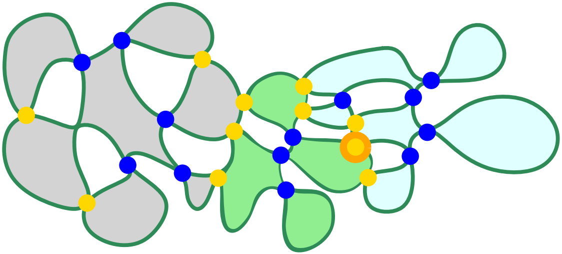

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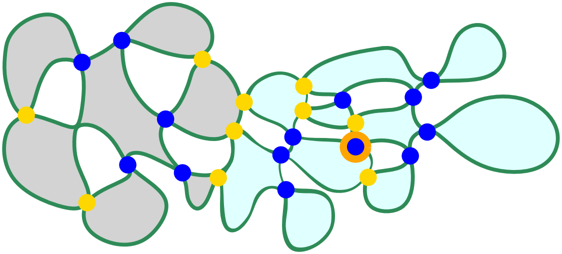

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(2) mixing			(1) cut-off convergence	
$(\mathcal{T}, (\mathscr{P}_t)_{t\geq 0})$	$\Longrightarrow$	$(h, (\Gamma_t))_{t\geq 0}$	$\Longleftarrow$	$(M, (P_t)_{t\geq 0})$
QDP on lattice		Continuum QDP		DP on RPM

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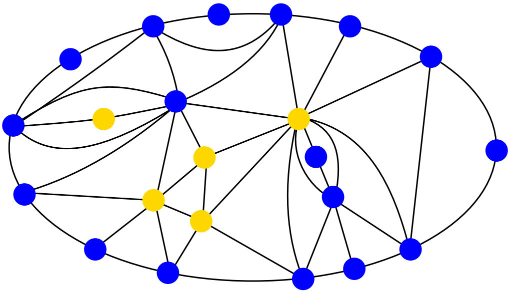
(2) mixing | | | (1) cut-off convergence

$$(\mathcal{T}, (\mathscr{P}_t)_{t\geq 0}) \quad \Longrightarrow \quad (h, (\Gamma_t))_{t\geq 0} \quad \Longleftarrow \quad (M, (P_t)_{t\geq 0})$$

QDP on lattice | Continuum QDP | DP on RPM



Thanks!