MAT 530, Homework 9, due Wednesday, 10/28

Please do questions from Hatcher posted on the course page, plus two problems below.

1. Let K be a finite graph, namely, a finite collection of vertices connected by a finite number of edges, where the topology is defined as follows. Let $V = \{v_1, v_2, \dots v_k\}$ be a finite set of points (*vertices*), equipped with discrete topology. Make K by attaching a finite number of *edges* to V: each edge is homeomorphic to a closed interval, the endpoints of each edge are glued to two vertices in V (not necessarily distinct), and the resulting space is equipped with the quotient topology. (A graph is a finite 1-dimensional cell complex.)

Show that the space obtained by contracting any edge of K is homotopy equivalent to K if the edge has distinct endpoints. More precisely, suppose that e is a closed edge of K (i.e. an edge taken together with its endpoints), and the endpoints of e are two different vertices of K. Then the quotient space K/e is homotopy equivalent to K.

Suppose K is a connected graph with k vertices and n edges. What space (up to a homeomorpism) will you get if you perform the operation described above as many times as possible?

2. For a continuous function $f : S^{n-1} \to X$, consider the space $X \cup_f D^n$ obtained by gluing the disk D^n to X along its boundary sphere via the function f. (As usual, the resulting space is given the quotient topology). Show that if $f, g : S^{n-1} \to X$ are two homotopic maps, then the spaces $X \cup_f D^n$ and $X \cup_g D^n$ are homotopy equivalent.
Hint: glue $D^n \times I$ to X by using the homotopy between f and g. Notice that $D^n \times I$ deformation retracts to $D^n \times \{0\} \cup S^{n-1} \times I$.