

## The Cantor Set

The Cantor set, by definition, is the complement in  $[0, 1]$  of the union of all intervals of the form  $(\frac{3k+1}{3^m}, \frac{3k+2}{3^m})$ ,  $m > 0$ . (If you never encountered the Cantor set before, check out the its Wikipedia page for pictures and basic information.)

Please prove the following properties of the Cantor set  $K$ :

- (a)  $K$  is closed.
- (b)  $K$  is nowhere dense.
- (c)  $K$  is totally disconnected. (See definition in 12'5.)
- (d)  $K$  has no isolated points.
- (e)  $K$  is uncountable.

The example of Cantor set shows that nowhere dense sets can be fairly complicated (and in some sense “big”). It is even possible to construct a Cantor-like set by removing intervals whose total length is less than 1, producing a nowhere dense set of positive “length” (Lebesgue measure).