MAT 530 Topology, Geometry I

Problem Set 6

due Friday, October 16

Problem 1. Let G be a group, $g \in G$, $g \neq e$. Show that G has a maximal normal subgroup not containing the element g. (It's ok if this maximal normal subgroup is $\{e\}$ in some cases.)

Reading assignment: please read Thm. 30.2 and Examples 3 and 4 of §30.

Please also do questions 2, 5, 6 ($\mathbb{R}_l$ part only), 10, 15 of §30.

Problem 2. Consider the space F of all bounded real-valued functions on $[0, 1]$ with the uniform topology, i.e the topology induced by the metric

$$d(f, g) = \sup_{[0,1]} |f - g|.$$

Is this space first-countable? Second-countable?