Problem 1. Consider two topological spaces $X$ and $X'$, defined as follows.

The space $X$ is a subspace of the standard $\mathbb{R}^2$, given in the polar coordinates by

$$X = \{(r, \theta) : 0 \leq r \leq 1, \theta = \frac{2\pi}{n} \text{ for some } n \in \mathbb{N}\}.$$ 

The space $X'$ is a subspace of $\mathbb{R}^\omega$, where the latter is the space of sequences with the metric topology induced by the metric

$$d((x_1, x_2, \ldots, x_n, \ldots), (y_1, y_2, \ldots, y_n, \ldots)) = \sup_i |x_i - y_i|.$$ 

(You do not have to check that this is indeed a metric.) Define $X' = \bigcup_{n \geq 1} Y_n$, where

$$Y_n = \{(y_1, y_2, \ldots, y_n, \ldots) : 0 \leq y_n \leq 1, y_i = 0 \text{ for all } i \neq n\}$$

(i.e a sequence is in $Y_n$ if its $n$-th coordinate is between 0 and 1, and all other coordinates vanish).

Show that $X$ and $X'$ are not homeomorphic (despite an obvious bijection between them).

Problem 2. Suppose $X$ is connected, $f : X \to \mathbb{R}$ is locally constant. (This means that every point $x \in X$ has a neighborhood $U \ni x$ such that $f$ is constant on $U$.) Show that $f$ is then a constant function.

Problem 3. Suppose $U \subset \mathbb{R}^n$ is open and connected. Show that $U$ is path-connected.

Please also do questions 11 of §17, 9, 13 of §18, 4 of §23, and 1, 2 of §24 of Munkres.

Required Reading: please read §25.