

MAT 511 Fundamental Concepts of Math

Problem Set 2

due Thursday, Sept 18

Please prove all your answers. Short and elegant proofs are encouraged but not required. You can use any method, but arguing by contradiction can be useful for a few questions below.

Problem 1. There are 400 people in the '12 class. Show that at least two of them share a birthday.

Problem 2. Some of 100 airports are connected by direct flights. Prove that there are two airports that have direct flights to the same number of destinations.

Problem 3. Prove that $\sqrt{3}$ is irrational.

Problem 4. Show that if 4 divides $a^2 + b^2$, then both a and b are even.

Problem 5. Construct the opposites for each statement. Please state your opposites in plain English (not in symbolic form); try to unravel them as much as you can. (“It is not true that blah” is not a very useful opposite for “blah”). You do not have to prove anything or to determine whether these statements are true or false.

(a) There is a natural number M such that for every natural number n , $1/n < M$.

(b) For every real number $\epsilon > 0$ there is a natural number M such that for all natural numbers $n > M$, $\frac{1}{n} < \epsilon$.

(c) For every real number $\epsilon > 0$, there is a natural number M such that if $m > n > M$, then $\frac{1}{n} - \frac{1}{m} < \epsilon$.

(d) $(\forall y)(\exists x)(\forall z)(xy = xz)$

Please also do questions 12(acd) of §1.5, 5(def) of §1.6 of *Eggen, Smith, St.Andre*.