

MAT 364 Topology

Problem Set 10

Solutions

1. The strip cut out in the middle is clearly a Möbius band (because you identify top and bottom after a twist, and the sides are not glued). The two leftover pieces give a diagram representing the Möbius band after one identifies their vertical sides along the arrows.

2. Although one can work with planar diagrams, the easiest way to see this is to recall that $K = P\#P$. Then we need to check that $T\#P\#P$ is homeomorphic to $P\#P\#P\#P$, but this follows from the identity $T\#P = P\#P\#P$ that we established earlier.

3. For the first diagram, the edges a - b can be replaced by one edge d (because they are glued in the same way). After that, you get a diagram for the Klein bottle.

For the second diagram, notice that c can be zipped up. After that, you get the diagram for the torus.

4.22 The answer is the same surface with a handle attached (ie the connected sum of the original surface with a torus).