MAT 364 Topology

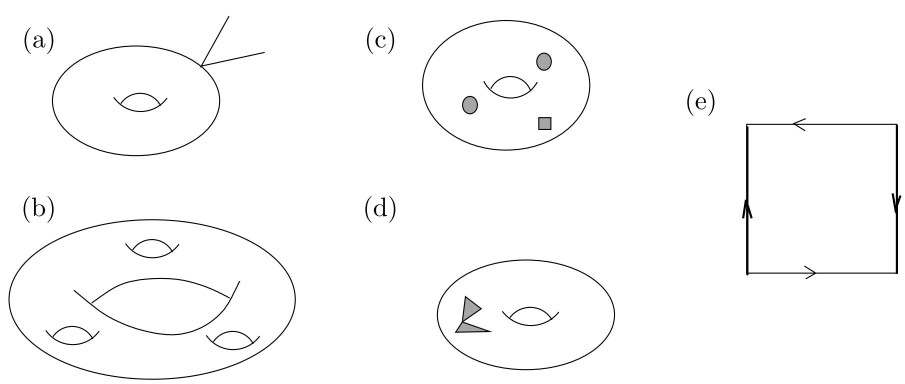
Problem Set 9

due Wednesday, November 10

Please do Exercises 4.2, 4.3, 4.4 from the book. In 4.2 and in Problem 1 below, there's nothing to "prove" – just state the answer and say a few words about how you saw it. In 4.3 and 4.4, give an explanation based on planar diagram and gluing sides of polygons, as we did in class.

Problem 1. (another experiment with the Möbius band) Notice that a cylinder has two boundary circles. How many boundary circles does a Möbius band have?

Problem 2. Determine which of the following spaces are surfaces with boundary, surfaces without boundary, or not surfaces at all. (Assume that they have the natural topology as subsets in the 3-space). Explain your answer.



(f) a cube
(g) the surface of a cube

In (c) and (d), the grey areas are cut-out holes. The holes are assumed to be open, i.e. their boundary remains in the space; in particular, the connecting point of the two deleted triangles in (d) remains in the torus. In (e), we identify the sides of the square as shown in the diagram, ie twist 180° before gluing each pair of opposite sides together.

Argue from the definition where every point is required to have a neigborhood homeomorphic to a disk (or half-disk for boundary points.) Please just give a heuristic explanation when saying whether such neighborhoods exist or not (you'd need a lot more mathematics to actually prove non-existence).

Problem 3. Find the connected sum of a torus and a disk. Also, describe (with proof) the connected sum of an arbitrary surface and a disk.