Problem 1. Let $X$ be a topological space. Suppose that $Y$ is a subset of $X$. We can define a topology on $Y$ as follows. If $\mathcal{T}_X = \{U_i\}$ is the topology on $X$, let $\mathcal{T}_Y$ to be the collection of all sets $U_i \cap Y$. (Notice that $U_i \cap Y$ are subsets of $Y$.)

Check that $\mathcal{T}_Y$ is indeed a topology on $Y$ (ie it satisfies the axioms). It is called a \textit{subspace topology}.

Problem 2. Suppose that $X$ is a topological space, $Y$ is a subset of $X$. As explained in Problem 1, $Y$ can be considered as a topological space (equipped with subspace topology). Prove that if $X$ is compact, and $Y$ is closed in $X$, then $Y$ is also compact.

Problem 3. Consider the set $\mathbb{R}^2$. For any two points $x = (x_1, x_2)$ and $y = (y_1, y_2)$, define the distance $d(x, y)$ by the formula

$$d(x, y) = |x_1 - y_1| + |x_2 - y_2|.$$ 

(a) Prove that $d$ satisfies the axioms for a distance, ie $(\mathbb{R}^2, d)$ is a metric space.

(b) Sketch the unit disk $D(0,1)$ centered at 0 for this metric.

Please also do Exercise 3.3 p. 40, Exercise 3.9 p. 43.