MAT 364 Topology

Problem Set 6

due Wednesday, October 13

Problem 1. Determine whether the following sets are compact or not. Prove your answers.
(a) $X = \{(x, y) \in \mathbb{R}^2 | x \geq 0, y \geq 0\},$ the graph of the function $y = \sin x$.
(b) $Y = \{(x, y) \in \mathbb{R}^2 | x \geq 0, y \geq 0\},$ the first quadrant .
(c) $Z = \{x^2 + y^2 + z^2 = 1\}$, a sphere in $\mathbb{R}^3$.
(d) $V = \{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots\}$.
(e) $W = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots\}$.
(d) $\mathbb{Q} \cap [0, 1]$, the set of rational numbers q such that $0 \leq q \leq 1$.

Problem 2. Suppose $A, B \subset \mathbb{R}^n$ are two compact sets. Prove that $A \cup B$ is compact. For more practice, please give three different proofs:
(a) from the definition with convergent subsequences
(b) from the characterization of compact sets in $\mathbb{R}^n$
(c) from the definition with open covers (to be discussed in Friday class).

Problem 3. We will prove in class on Friday that if $f : D \to R$ is a continuous function, and X is a compact set in D, then $f(X)$ is compact. (Here $D \subset \mathbb{R}^n$, $R \subset \mathbb{R}^m$ as usual). Show that
(a) if $f : D \to R$ is continuous, and X is closed in $\mathbb{R}^n$, $X \subset D$, then $f(X)$ doesn't have to be closed.
(b) if $f : D \to R$ is continuous, and X is a bounded subset of D, then $f(X)$ doesn't have to be bounded.
(Give counterexamples and justify all the properties you need.)