MAT 364 Topology

Problem Set 4

due Wednesday, September 29

Problem 1. Are the following functions continuous? Are they homeomorphisms? Explain.
(a) $f : [-1, 1] \to [0, 1]$, $f(x) = |x|$
(b) $g : [0, 1) \to C$, where $C = \{(x, y) : x^2 + y^2 = 1\}$ is the unit circle in the plane, and the function g sends $x \in [0, 1)$ to the point on the circle corresponding to the angle $2\pi x$.
(If you want a formula, $g(x) = (\cos 2\pi x, \sin 2\pi x)$).
Although we discussed the second example in class, please write a detailed explanation for your answers.

Problem 2. We know from claculus/analysis that the following functions $f, g : \mathbb{R} \to \mathbb{R}$ are not continuous. Show this using the topological definition of continuity ("f^{-1}(open) is open").

(a) $f(x) = \begin{cases} 3x, & x \neq 2 \\ 0, & x = 2 \end{cases}$

(b) $g(x) = \begin{cases} \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Problem 3. Let $A \subset \mathbb{R}^m, B \subset \mathbb{R}^n$ be some spaces. Recall that A is homeomorphic to B if there exists a function $f : A \to B$ which is a homeomorphism.
(a) Prove that if A is homeomorphic to B, then B is homeomorphic to A.
(b) Prove that if A is homeomorphic to B, and B is homeomorphic to C, then A is homeomorphic to C.
(Please give a careful argument; saying that A and B are "topologically the same" is not a proof.)

Problem 4. Do question 2.29, ie prove that the definition of connectedness we gave in class is equivalent to the definition 2.27 in the book.

Please also do question 2.31 from the book.