MAT 364 Topology

Problem Set 12

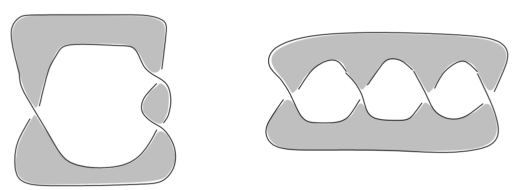
due Friday, December 10

Problem 1. Consider the graph whose vertices are vertices of a regular octagon, and the edges are given by the octagon's diagonals (each vertex is connected by diagonals to all the 5 vertices that are not adjacent to itself). Prove that this graph is not planar.

Problem 2. A while ago, we saw that the Möbius band whose boundary is glued up with a disk is the projective plane.

Give a different proof of this fact, using the following strategy. First, explain why the Möbius band glued up with a disk gives a non-orientable surface without boundary. Second, compute its Euler characteristic. (You can use the fact that χ(Möbius)=0, see Hw 11.) Using classification of surfaces, conclude that the result must be a projective plane.

Problem 3. Identify the following surfaces with boundary (as connected sums of tori or projective planes with a certain number of holes) by using the Euler characteristic and the classification of surfaces.



Problem 4. Decide which of the letters B, E, I, L, O, P, T are homeomorphic, and prove your answer. Assume that letters are thin (ie can be represented by graphs), and drawn as simply as possible (for instance, I is just a segment of a straight line). For letters that are homeomorphic to one another, briefly explain why (you don't have to write formulas or prove continuity). To prove that two letters aren't homeomorphic, use the Euler characteristic or a connectedness argument, as follows. If there is a homeomorphism f between spaces X and Y, and $x \in X$ is a point mapped to $f(x) \in Y$, then $X - \{x\}$ must be homeomorphic to $Y - \{f(x)\}$ (check this!). If x can be chosen so that one of the spaces $X - \{x\}$ and $Y - \{f(x)\}$ is necessarily connected, and the other is not, you get a contradiction.