MAT 364 Topology

Problem Set 11

due Friday, December 3

For all questions in this problem set, you may take for granted that the Euler characteristic (computed as # faces -#edges +# vertices) is independent of the way the surface is divided into polygonal faces.

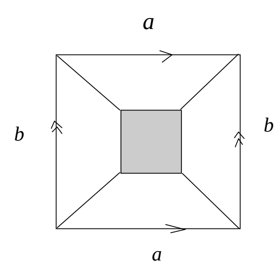
Problem 1. Compute the Euler characteristic of

(a) the cylinder

(b) the Möbius band.

You can divide them into polygons any way you like, but make sure you don't forget edges running along the boundary.

Problem 2. (a) Compute the Euler characteristic of the torus with a hole cut out. Use the diagram shown below.



(b) Suppose S is a surface with the Euler characteristic $\chi(S) = m$. What happens to the Euler characteristic when you cut a hole in S? Find (with proof) the Euler characteristic of the surface with boundary obtained from S by cutting k holes out.

Problem 3. Investigate what happens to the Euler characteristic when you take connected sums. If $\chi(S_1) = a$ and $\chi(S_2) = b$, what is $\chi(S_1 \# S_2)$? Prove your answer.

Hint. Recall that the connected sum is obtained by cutting small disks out of both surfaces and gluing them together. In the previous problem, you figured out what happens to χ when you cut a hole out. When S_1 with a hole and S_2 with a hole are glued together, each edge on the boundary of the hole in S_1 would be glued to a similar edge in S_2. Take this into account when counting edges in the resulting surface.

For partial credit, feel free to work with special cases if the general picture seems scary (although it's really not hard!).