Problem 1. Prove that if you glue two Möbius bands along their boundary circles, the result is the Klein bottle. **Hint:** cut the Klein bottle diagram along the dashed lines as shown in the picture.

Problem 2. Prove that $T \# K$ is homeomorphic to $K \# K$. Here, $T$ is the torus, and $K$ is the Klein bottle as usual.

Problem 3. Identify, with proof, the surfaces shown below: determine to what familiar surface (or connected sums) each of them is homeomorphic.

Please also do Exercise 4.22 from the book.