MAT 364 Topology

Exam II checklist

Below is the list of topics, definitions and theorems that we discussed so far. For the exam, you are required to know all the definitions. In your solutions of exam problems, you can refer to any of the theorems and facts we proved in class. You will not be asked to reproduce any of those proofs, but familiarity with the techniques is expected and will be useful.

General topological spaces

Axioms of a topological space, open and closed sets
Continuous functions between topological spaces
Compactness, connectedness, Hausdorff property in general topological spaces
Theorems that can be generalized from the Euclidean case: if f is continuous, and X is connected, then $f(X)$ is connected; same for compact sets, etc.
Facts that are no longer true: compact is not the same as closed and bounded!
A sequence can't have two limits in Hausdorff spaces (but in general might)

Metric spaces

Axioms for a metric space
Examples; pictures of balls and "circles" in various metrics
Topology induced by the metric, open and closed sets
Using the triangle inequality; Theorem: $D(a, r)$ is open
Examples of metrics that induce the same/different topology on the same set
Continuity in metric spaces
Sequences and limits in metric spaces
Metric spaces are Hausdorff

Surfaces

Definition, examples; checking that a space is/isn't a surface; surfaces with, without boundary
Homeomorphisms between surfaces vs. embeddings into 3-space
Torus, sphere, projective plane, Klein bottle; genus g surfaces; Möbius band
Orientable and non-orientable surfaces
Connected sums and their properties
Planar diagrams; topology defined by a planar diagram
Cut-and-paste technique; examples – $P\#P = K$, $T\#P = P\#P\#P$
Theorem: let M be a compact, connected surface without boundary. If M is orientable, then M is homeomorphic to a sphere or a connected sum of tori; if M is non-orientable, then M is homeomorphic to a connected sum of projective planes.