MAT 360 Geometry

Homework 8

due Thursday, Apr 11

Problem 1. Prove that any two congruent segments can be superimposed by some rotation. **Hint:** think two reflections.

Problem 2. Let l_1 and l_2 be two lines that intersect at the point O; R_1, R_2 denote reflections about these lines. In class, we discussed that the composition of these reflections is the same as the rotation about O by twice the angle between the lines. This may cause some confusion: indeed, you have to pay attention to the order of reflections (which one is performed first), to the direction of rotation (clockwise or counterclockwise), and to the choice of the angle (the lines cut the plane into 4 angles). We saw that, more precisely,

$$R_2 \circ R_1 = Rot_{O,2\alpha}, \tag{1}$$

where in the composition R_1 is performed first, and α stands for the angle measured *from* the line l_1 to the line l_2 *counterclockwise*. By convention, α is always taken to be less than 180° (but it can be acute, right or obtuse). The purpose of this problem is for you to get used to all these subtleties.

Now, let R_x be the reflection about the x-axis, R_{xy} the reflection about the line $x = y$.

(a) Using the formula above, what do you get for $R_{xy} \circ R_x$? Prove your answer directly: check where a few "nice" points go, and use the fact that an isometry is determined by its effect on any 3 non-collinear points.

(b) Repeat part (a) for the composition $R_x \circ R_{xy}$.

(c) Prove that $R_x \circ R_{xy}$ is the *inverse* of $R_{xy} \circ R_x$ (in the sense of MAT 200), i.e. $R_x \circ R_{xy}$ undoes the effect of $R_{xy} \circ R_x$. Therefore, if the first composition is a *counterclockwise* rotation by a certain angle, the second must be the *clockwise* rotation by the same angle. How does this fit with formula (1), where we only consider counterclockwise rotations? Explain.

Problem 3. Prove that a composition of a rotation and a translation is a rotation. (This is true for any order of the rotation/translation; for concreteness, you can assume that the rotation is performed first.)

Problem 4. Prove that the composition of any two central symmetries is a translation. More precisely, $S_B \circ S_A = T_{2\overrightarrow{AB}}$, where S_X denotes the symmetry about point X, and T is the translation by the vector indicated.

Problem 5. Prove that the composition of a central symmetry about X and a reflection about l is a glide reflection, provided that the point X does not lie on l. **Hint:** decompose the central symmetry into a composition of two reflections.

Problem 6. Let X be a point on the line l, and consider the composition of the reflection about l with a rotation about X (by an arbitrary angle). Determine whether the resulting isometry is a translation, rotation, reflection or glide reflection. Prove your answer.