MAT 360, Geometry

Spring 2011

Final Exam

Name:_____

This is a closed book, closed notes test. No consultations with others. Calculators are not allowed.

Please turn off and take off the desk cell phones, pagers, etc. Only the exam and pens/pencils should be on your desk.

Please explain all your answers, show all work, and give careful proofs. Answers without explanation will receive little credit.

The problems are not in the order of difficulty. You may want to look through the exam and do the easier questions first.

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1. (a) Show that a figure similar to a rhombus is a rhombus.

(b) Is it true that any two rhombi are similar? Give an "if and only if" condition for two rhombi to be similar, and prove your answer.

Several different proofs are possible, you can use any method.

2. Let S be the central symmetry about the origin, and T the translation by vector $(2, 0)$ in the (x, y) plane.

(a) Will the compositions $S \circ T$ and $T \circ S$ be isometries? Explain your answer.

(b) Is it true that $S \circ T = T \circ S$? Explain.

(c) Describe explicitly the transformation $T \circ S$; prove your answer. (Determine the corresponding isometry type and all the relevant parameters, such as the angle and center for rotations, the axis of reflection and the translation vectors for glide reflections, etc.)

There are a few possible ways to approach this problem, you can use any method.

3. Given an angle with vertex A and a point M inside the angle, construct a line that passes through M and intersects the sides of the angle $\angle A$ at points B and C such that

$$\frac{AB}{AC} = \frac{2}{5}.$$

Use a compass and a straightedge only; justify your construction.

(You can use, without a detailed description, the following elementary constructions: segment and angle bisection, raising a perpendicular at a point on the line, dropping a perpendicular from a point not on the line, constructing segments and angles congruent to given ones.)

4. Circle the correct answers to questions below. **No explanations are needed.**

(a) Any similarity mapping can be written as a composition $R \circ H$, where R is a rotation, translation, reflection or glide reflection, and H is a homothety.

TRUE FALSE

(b) For any two given straight lines l and m, one can find a homothety that maps l to m.

TRUE FALSE

(c) For any two given straight lines l and m, one can find a rotation that maps l to m.

TRUE FALSE

(d) (CIRCLE ALL THAT APPLY)
A composition $T \circ R_1 \circ R_2$ of two reflections R_1, R_2 and a translation T can be, depending on parameters,

(i) a rotation (ii) a translation (iii) a reflection (iv) a glide reflection

(e) The sides of any triangle are commensurable.

TRUE FALSE