MAT 360 Geometry

Homework 9

due Thursday, Nov 10

Problem 1. Prove that the composition of any two central symmetries is a translation. More precisely, $S_B \circ S_A = T_{2\overrightarrow{AB}}$, where S_X denotes the symmetry about point X, and T is the translation by the vector indicated.

Problem 2. Prove that the composition of a central symmetry about X and a reflection about l is a glide reflection, provided that the point X does not lie on l. **Hint:** decompose the central symmetry into a composition of two reflections.

Problem 3. Let X be a point on the line l, and consider the composition of the reflection about l with a rotation about X (by arbitrary angle). Determine whether the resulting isometry is a translation, rotation, reflection or glide reflection.

Problem 4. A similarity mapping with coefficient $k > 0$ is a plane transformation that scales all distances by the factor k, i.e. S is a similarity mapping of for any two points A, B we have $S(A)S(B) = kAB$. (We will soon discuss in detail what the last equality means for arbitrary k; for now, you may assume that k is an integer.

Prove that the composition of a similarity mappings with coefficients k and l is a similarity mapping. What is its coefficient?

Problem 5. We discussed in a class that a basic example of a similarity mapping is given by homothety (scaling up from one point by some fixed coefficient k), and that composing a homothety with an arbitrary isometry produces a similarity mapping (compare with Problem 4).

Using this, prove that any two circles are similar. (Recall our definition of similar figures: you have to show that for any two circles, there exists a similarity transformation that sends one to the other.)

Please also do Question 338a on p.127. (Read sections 144-146 or wait until we discuss the relevant material on Tuesday.)