

MAT 360 Geometry

Homework 7

due Thursday, Oct 27

Problem 1. Prove that any isometry maps a circle into a circle of the same radius. (An isometry is a distance-preserving transformation of any kind. Please don't just assume that it has to be a rotation, a reflection, etc. You have to show that if you apply an arbitrary isometry to a given circle, the image will always be a circle, as opposed to some weird shape.)

Problem 2. In the xy -plane, let R_x be the reflection about the x -axis, R_y the reflection about the y -axis, R_o the rotation about the origin by 90° counterclockwise.

(a) Describe $R_x \circ R_y$ and $R_y \circ R_x$. Are they the same?

(b) Describe $R_x \circ R_o$ and $R_o \circ R_x$. Are they the same?

All of the above compositions will be reflections, rotations or translations. "Describe" means determine which type they are, and give the axis for reflections, the center and angle for rotations, or the vector for translations, as appropriate.

Problem 3. Consider the composition of a reflection R about the line l with the translation T by a vector \vec{AB} parallel to l .

(a) Show that the order in which the translation and the reflection are performed does not matter here, i.e. $R \circ T = T \circ R$. Explain why the resulting transformation is an isometry.

(b) Show that $R \circ T$ **cannot** be represented as a rotation, reflection, or translation.

Problem 4. Prove that a triangle with (at least) two distinct axes of symmetry must be equilateral. How many axes of symmetry does an equilateral triangle have? Does there exist a triangle with exactly two distinct axes of symmetry? Explain your answers.

Problem 5. How many axes of symmetry can a quadrilateral have? Consider all cases, prove your answer.