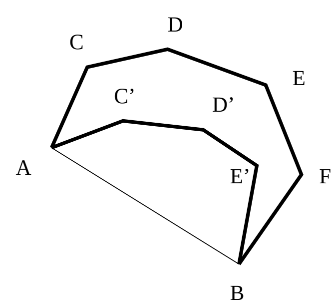
MAT 360 Geometry

Homework 3

due Thursday, Sept 22

Problem 1. (a) Let $ACDEF...B$ be a *convex* broken line, X a point inside the polygon $ACDEF...B$. Show that $AX + XB < AC + CD + DE + ...$, i.e. $AX + XB$ is shorter that the sum of all segments of the broken line. (The broken line can have an arbitrary number of segments.)

(b) More generally, let $ACDEF...B$ be a *convex* broken line, $AC'D'E'...B$ is another broken line contained inside it. Show that the sum of segments $AC + CD + DE + ..$ of the *outer* broken line is greater than the sum of segments of the *inner* broken line, $AC' + C'D' + D'E' + ...$ See picture; the two broken lines can consist of arbitrary (and not necessarily the same) number of segments.



(c) What happens if the outer broken line is not convex? What happens if the inner broken line is not convex? Justify your answer.

Problem 2. Four houses A, B, C, D form vertices of a square. The residents would like to dig a well at a point W such that the sum of distances $AW + BW + CW + DW$ from all the houses to the well is the smallest possible. Where should they dig the well? Prove your answer.

Problem 3. Prove theorems from section 53 in the book. (Argue by contradiction, use the theorem in section 52.)

Please also do questions 90, 91, 95 from the book.