MAT 360 Topology

Problem Set 10

due Thursday, Nov 17

In this problem sheet, $H_{P,k}$ stands for homothety with center P and coefficient k, as usual.

Problem 1. In class, we proved that any similarity transformation S with coefficient k can be written as a composition $S = I \circ H_{O,k}$ of some isometry and a homothety with coefficient k and arbitrarily chosen center O. Here, homothety is performed first, isometry second.

Using the same method, prove that any similarity transformation S can also be written as a composition $S = H_{O,k} \circ I'$, where isometry I' is performed first, homothety $H_{O,k}$ second. (Notation I' is used to show that for a given S, isometries for the two compositions may be different).

Problem 2. Prove that any similarity transformation with coefficient k has an inverse (in the sense of MAT 200), and that the inverse is also a similarity transformation. What is its coefficient?

Problem 3. Prove that a figure similar to a circle is a circle, i.e. any similarity transformation maps every circle to a circle. (Note that this *does not* repeat a question from the last homework.) **Hint:** $S = I \circ H_{O,k}$.

Problem 4. (a) Show that two homotheties with different centers A, B and the same coefficient differ by rotation or translation, i.e. $H_{A,k} = R \circ H_{B,k}$, where R is some rotation or translation that depends on the homotheties.

(b) More precisely, prove that $H_{A,k} = T \circ H_{B,k}$, where T is the translation by vector $(k-1)\vec{AB}$.

(Part (a) follows from part (b), but is much easier to prove.)