Reference Page

Fourier series representation of a periodic function $f(x)$ of period $2a$:

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{\pi n x}{a}\right) + b_n \sin\left(\frac{\pi n x}{a}\right),$$

where

$$a_0 = \frac{1}{2a}\int_{-a}^{a} f(x)\,dx, \quad a_n = \frac{1}{a}\int_{-a}^{a} f(x)\cos\left(\frac{\pi n x}{a}\right)\,dx, \quad b_n = \frac{1}{a}\int_{-a}^{a} f(x)\sin\left(\frac{\pi n x}{a}\right)\,dx.$$

Fourier integral representation of $f(x)$ satisfying appropriate conditions:

$$f(x) \sim \int_0^{\infty} (A(\lambda)\cos(\lambda x) + B(\lambda)\sin(\lambda x))\,d\lambda, \quad -\infty < x < \infty,$$

where $A(\lambda) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos(\lambda x)\,dx$, $B(\lambda) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin(\lambda x)\,dx$.

For the vibrating string problem

$$\frac{\partial^2 u}{\partial x^2}(x,t) = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}(x,t), \quad 0 < x < a, \quad t > 0$$

with fixed ends $u(0,t) = u(a,t) = 0$ and initial data $u(x,0) = f(x)$, $\frac{\partial u}{\partial t}(x,0) = g(x)$, D'Alembert solution is given as a sum of two waves $u(x,t) = \psi(x+ct) + \phi(x-ct)$, where

$$\psi(x+ct) = \frac{1}{2}(\tilde{f}_{odd}(x+ct) + \tilde{G}_{even}(x+ct)), \qquad \phi(x-ct) = \frac{1}{2}(\tilde{f}_{odd}(x-ct) - \tilde{G}_{even}(x-ct)),$$

with $G(x) = \frac{1}{c}\int_0^x g(y)\,dy$.

Potential equation in polar coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} = 0.$$