MAT 341
Midterm I checklist

Fourier series:

Memorize the formulas for the Fourier series of a periodic function f with period $2a$ and know how to use them. Know how to derive and use formulas for the sine/cosine series for odd/even functions. (No need to memorize these additional formulas: if you understand odd and even functions well, you should clearly see how the sine/cosine series are related to the main formula. You'll get confused if you memorize too many similar formulas.)

While you will not have to compute long and difficult integrals on the test, you should be able to use basic integration techniques (such as integration by parts). You should also be very familiar with odd and even functions (properties, graphs, integration). You should also know that Fourier series for linear combinations of sines and cosines (with matching periods!) such as $3\sin\pi x - 2\cos 7\pi x + \sin 4\pi x$ are given by these combinations themselves.

You should know convergence theorems (Theorem from 1.3, Theorem 2 from 1.4 and Theorems 3,4,5 which follow) and have an intuitive understanding of uniform convergence vs convergence for every x individually. When working with extensions, always draw the periodic extension on the whole interval $(-\infty, \infty)$ and be careful with endpoints of the given interval, otherwise you'll miss the possible jumps there.

Questions may include:

Computing Fourier series:

- Determine which of the given functions are periodic, find period
- Sketch graphs of the periodic extenstion of a function given on $(-a, a)$, and odd/even periodic extensions of a function on $(0, a)$
- Compute certain Fourier coefficients for a given function; write an explicit expression for a given Fourier coefficient but do not compute[1]
- compute the Fourier series for a periodic function or for a periodic extension of a function given on an interval $(-a, a)$
- Use odd/even extensions to find sine/cosine series for a given function on $(0, a)$

Convergence of Fourier series:

- determine to what value the Fourier series of the function converges at a given point; explain your answer.
- determine whether the convergence is uniform; explain your answer.

Practice: 1.1 questions 1, 2; 1.2 questions 1, 5, 7, 10; 1.3 questions 2, 3, 5, 6; 1.4 questions 1, 3; p.118 questions 1, 7cde, 10, 11, 12, 13, 14, 15, 30, 30af.

The heat equation:

You should be familiar with the setup of the heat equation (differential equation itself, boundary conditions, intitial conditions), with all the steps required to solve it (see summary at the end of 2.5), and all the terminology (steady-state, transient solutions). It's important to remember that the equation for the transient solution should be *homogeneous*, so if the original equation or boundary conditions are non-homogeneous (for example, there's a constant added to the equation), the extra terms will probably cancel.

Questions may include:

- Find the steady-state solution for a given equation with given boundary conditions. The equation may include extra terms such as generation (??); you will need basic MAT 303 skills to solve ordinary differential equations. Boundary conditions may be of different types, too.
- Explain the physical meaning of the steady-state solution (it's a "stable" solution after a lot of time has passed and the temperature distribution is not changing with time anymore).
- Find the equation and the boundary and initial conditions for the transient solution for a given problem (do not solve). Again, the equation may include extra terms for generation, and different boundary conditions may appear.

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[1] make it as explicit as possible without computing: for example, if the function f is given by $f(x) = x^2$ for $0 < x \leq 1$, $f(x) = x$ for $1 < x < 2$, and you are working with $\int_0^2 f(x)\cos 3\pi x dx$, you should expand it as $\int_0^1 x^2 \cos 3\pi x dx + \int_1^2 x \cos 3\pi x dx$

- Given a heat equation problem with boundary conditions and initial conditions, go through all the steps to solve the equation. You will *only* be required to solve the standard fixed end temps or insulated bar questions (2.3 or 2.4), with specific initial and boundary conditions. You should be able to produce the complete solution *without* the prompts for each step.

Practice: 2.2 questions 1–8; 2.3 questions 5–8; 2.4 questions 1–5; p. 205 questions 1-3, 5-7, 10

Sturm-Liouville problem:

You should be familiar with the statement of the problem, the eigenvalues/eigenvectors terminology, and the orthogonality relation. Only the basic problem as in 2.7 equations (1)-(3) will be on the test (the more general Sturm-Liouville problem, equations (5)-(7) is not on the test).

Questions may include:

- recognize which of the given equations and boundary conditions are a Sturm-Liuoville problem.
- find eigenvalues and eigenvectors for a given Sturm-Liouville problem.
- state the orthogonality relation for the given problem.

Practice: 2.7 questions 2, 3, 4, p.209 28, 29 (28 and 29 have an extra x, but you should still be able to solve it).