MAT 320 Introduction to Analysis

Homework 9
due WEDNESDAY, November 15

This is a shorter homework because it’s due on Wednesday, not Friday as usual.

1. Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous, $\lim_{x \rightarrow \infty} f(x) = 0$, and $\lim_{x \rightarrow -\infty} f(x) = 0$. Show that $f$ is bounded on $\mathbb{R}$ and attains a maximum or a minimum. Does it necessarily attain both its maximum and its minimum?

2. Let $f : [0,1] \rightarrow \mathbb{R}$ be a continuous function with the property that for every $x \in [0,1]$ there exists $y \in [0,1]$ such that $|f(y)| \leq \frac{1}{2} |f(x)|$. Show that there exists $c \in [0,1]$ such that $f(c) = 0$. **Hint:** use the Maximum-Minimum theorem.

3. Let $f : [0,1] \rightarrow \mathbb{R}$ be a function which takes on each of its values exactly twice. Show that $f$ cannot be continuous. **Hint:** arguing by contradiction, assume that $f$ is continuous, and consider the points where it attains a maximum and a minimum.

4. Suppose that $f : [0, +\infty) \rightarrow \mathbb{R}$ is continuous, and $\lim_{x \rightarrow \infty} f(x) = 0$. Show that $f$ is uniformly continuous.