1. Let $S = \left\{ \frac{1}{n} - \frac{1}{m} \mid n, m \in \mathbb{N} \right\}$. Find $\inf S$, $\sup S$ (if they exist). Explain your answer.

2. Let functions $f, g : [0, 1] \to \mathbb{R}$ be both bounded. Show that
   \[ \sup_{x \in [0,1]} (f(x) + g(x)) \leq \sup_{x \in [0,1]} f(x) + \sup_{x \in [0,1]} g(x). \]

   Give an example where $\sup_{x \in [0,1]} (f(x) + g(x))$ is not equal to $\sup_{x \in [0,1]} f(x) + \sup_{x \in [0,1]} g(x)$ (for some functions $f, g$).

   What would be the corresponding inequality for infimums?

3. As usual, for a real number $a$ we denote $V_\varepsilon(a) = \{ x \mid |a - x| < \varepsilon \}$.

   Find $\bigcap_{n \in \mathbb{N}} V_{1/n}(a)$.

   (Prove your answer using the Archimedean Property.)

4. Let $I_n = (-n, +\infty)$, $J_n = (-\infty, -n)$ for $n \in \mathbb{N}$. Find $\bigcap_{n \in \mathbb{N}} I_n$ and $\bigcap_{n \in \mathbb{N}} J_n$.

5. Find the binary representation of the following numbers:
   (a) $\frac{2}{3}$
   (b) $0.625$

   Explain how this relates to nested intervals.