1. Suppose functions $f(x), g(x)$ are defined in a neighborhood of $c$ (except perhaps for $x = c$), and $\lim_{x \to c} f = +\infty$ and $\lim_{x \to c} g = -\infty$. Give examples where
   (a) $\lim_{x \to c} (f + g) = -3$;
   (b) $\lim_{x \to c} (f + g) = +\infty$;
   (c) $\lim_{x \to c} (f + g) = -\infty$;
   (d) the function $(f + g)$ is bounded in a neighborhood of $c$, but $\lim_{x \to c} (f + g)$ does not exist.

2. Suppose $\lim_{x \to +\infty} f(x) = 0$, and the function $g(x)$ is bounded on $\mathbb{R}$. Arguing from definitions, show that $\lim_{x \to \infty} fg = 0$.

3. Let $f(x) = \sqrt{x} + 3$.
   (a) Find $\alpha \in \mathbb{R}$ such that $f(x) > 100$ for all $x > \alpha$. (You don’t have to find the best possible $\alpha$, but you have to show that your $\alpha$ works.)
   (b) Show that $\lim_{x \to +\infty} f(x) = +\infty$

4. (a) The function $f(x) = (x - 1)(x - 2)(x - 3)(x - 4)(x - 5)$ has five roots in the interval $[0, 7]$. If the Bisection method is applied on this interval, which of the roots is located?
   (b) Same question for $g(x) = (x - 2)(x - 3)(x - 4)(x - 5)(x - 6)$ on the interval $[0, 7]$.

5. Suppose that the function $f : \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$ and that $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to +\infty} f(x) = 0$. Prove that $f$ is bounded on $\mathbb{R}$ and attains either a maximum or minimum on $\mathbb{R}$. Give an example to show that both a maximum and a minimum need not be attained.

6. Let $f, g : [0, 1] \to \mathbb{R}$ be two continuous functions. Suppose that for a sequence $(x_n)$ of real numbers $x_n \in [0, 1]$ we have
   \[ f(x_n) = g(x_n) + \frac{1}{n}. \]
   Prove that $f(y) = g(y)$ for some $y \in [0, 1]$.
   **Hint:** Use Bolzano–Weierstrass theorem to choose a convergent subsequence out of $(x_n)$.