MAT 319 - Spring 2016 Quiz 1 Solution

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Problem:
Using the definition of the limit only (i.e. the so-called ε-N definition), prove that if $x_n \to -1$ and $y_n \to 3$, then $x_n + y_n \to 2$ for any two such sequences of real numbers $\{x_n\}$ and $\{y_n\}$. You have **10 MINUTES**.

Remarks:

- Do literally not use any theorem. Use the definition ONLY in your proof.
- You may use the back of the page.
- If you have any questions, ask.

Answer: Since $x_n \to -1$ and $y_n \to 3$, then by definition:

$$\forall \varepsilon > 0, \exists N_1 \in \mathbb{N}, \forall n > N_1 : |x_n - (-1)| < \varepsilon/2$$

$$\forall \varepsilon > 0, \exists N_2 \in \mathbb{N}, \forall n > N_2 : |y_n - 3| < \varepsilon/2$$

Picking $N = \max\{N_1, N_2\}$, then both inequalities are valid for any $n > N$, and therefore, given an arbitrary $\varepsilon > 0$, we have that for any $n > N$:

$$|(x_n + y_n) - 2| = |(x_n - (-1)) + (y_n - 3)| \leq |x_n - (-1)| + |y_n - 3| < \varepsilon/2 + \varepsilon/2 = \varepsilon$$

Therefore: $\forall \varepsilon > 0, \exists N = \max\{N_1, N_2\} \in \mathbb{N}, \forall n > N : |(x_n + y_n) - 2| < \varepsilon$; i.e., $x_n + y_n \to 2$ by definition.

(You'll have obtained 0 points if your solution was wrong, including a wrong statement of the definitions, 1 point if you wrote the correct definition but had nothing else right, or made major mistakes, 2 points if your mistakes were minor: for instance, you forgot to mention that the "N's" for the sequences should be different, missed a step in the inequalities, or did not write the complete definition, and finally 3 points if everything was correct.)