MAT 319
Homework 9
due Wednesday, April 6

Question 1. Do exercise 18.1 from the textbook, and explain how this shows that any continuous function attains its minimum on $[a, b]$. (We have proved that any continuous function attains its max on $[a, b]$; you can use this fact as known.)

Question 2. For a different proof that any continuous function attains its minimum on $[a, b]$, use the idea of the proof of Theorem 18.1 (p.133) and repeat all the steps with the necessary adjustments. (In particular, you will be proving that there is a point $x_0 \in [a, b]$ such that $f(x_0) = \inf f$.)

Question 3. Let function f be continuous at x_0. Suppose that $f(x_0) < c$ for some number c. Show that there is $\delta > 0$ such that $f(x) < c$ whenever $x_0 - \delta < x < x_0 + \delta$ and $x \in dom(f)$.

Question 4. (a) Let (s_n) be a convergent sequence, such that $\lim s_n < a$ for some number a. Show that there is $N \in \mathbb{N}$ such that $s_n < a$ whenever $n > n$. Argue directly from the definition of the limit.

(b) Suppose that $s_n \geq a$ for all n, and the sequence (s_n) converges. Show that $\lim s_n \geq a$. Use part (a).

(c) Suppose that $s_n \leq b$ for all n, and the sequence (s_n) converges. Show that $\lim s_n \leq b$.

(Notice that (b) and (c) together show that the limit of a sequence of points in $[a, b]$ must lie in $[a, b]$, a fact that we used in the proof of Theorem 18.1.)

(d) Will the statement of (b) remain true if you replace $\geq$ by $>$ everywhere? Will (c) remain true if you replace $\leq$ by $<$? Explain your answers.

Please also do questions 18.4, 18.6 from the book.