MAT 319
Homework 6
due Wednesday, March 9

Please do questions 14.4 and 15.6(a),(b). In 15.6(b), please give a complete proof (without using any non-assigned exercises or theorems not discussed in class). Hint: use comparison strategy. In 14.4, please justify everything.

Question 1. Prove the following version of the Root Test:
Suppose $\sum a_n$ is a series with non-negative terms, such that there is a limit

$$\lim \sqrt[n]{a_n} = L.$$

Prove:
(a) if $L < 1$, the series $\sum a_n$ converges.
(b) if $L > 1$, the series $\sum a_n$ diverges.
The book discusses a more complicated version of the Root test. You're welcome to read the book, but please give a *complete* proof for the case given above. This is similar (but easier!) than the proof of the Ratio Test given in class (see notes).

Question 2. Prove the following theorem:
Suppose $\sum a_n$, $\sum b_n$ are two series with positive terms. Suppose that there exists a limit $L = \lim \frac{a_n}{b_n}$, and $L \neq 0$.
Show that (a) if the series $\sum b_n$ converges, then $\sum a_n$ also converges, and (b) if $\sum b_n$ diverges, then $\sum a_n$ also diverges.
(This is sometimes called "limit comparison test". We proved it in class for a specific example. Use the same strategy to prove the general version.)
(c) Which part of your proof uses the condition $L \neq 0$?

Question 3. (a) Do question 14.8.
(b) Also, show that if $\sum a_n$ and $\sum b_n$ are two convergent series with non-negative terms, then $\sum a_n b_n$ also converges.
(c) If $\sum a_n = A$ and $\sum b_n = B$, is it true that $\sum a_n b_n = AB$? Justify your answer.