MAT 319
Homework 5
due Monday, February 29

Question 1. Exercise 11.10(a), page 77 in the textbook. Please prove your answer: show carefully that you've found ALL subsequential limits. Read the proof of theorem 11.2(i) and use this theorem.

Question 2. Let (x_n) and (y_n) be two sequences, both converging to a. Make a new sequence, (z_n), by alternating the terms x_n and y_n: $(z_n) = (x_1, y_1, x_2, y_2, x_3, y_3, \dots)$. More formally, we can define (z_n) by $z_{2n-1} = x_n$, $z_{2n} = y_n$.

Prove that (z_n) converges to a.

Your proof will be somewhat easier if you use the theorem from "More on Limits" notes, although you can also argue from the standard definition.

Question 3. Suppose the sequence (s_n) converges to L, and the sequence (t_n) differs from (s_n) only in finitely many terms. (For example, t_2, t_7, t_9, t_{13} might be different from s_2, s_7, s_9, s_{13}, and for all the other terms $s_n = t_n$.)

Prove that (i) if the sequence (s_n) converges, (t_n) converges to the same limit, and (ii) if (s_n) diverges, then (t_n) diverges as well.

This is a useful statement because it allows to change a (finite) bunch of terms in a sequence without affecting convergence.

Question 4. Prove part (iii) of Theorem 11.2 (p.68 of textbook); show that your subsequence can be chosen to be monotonic.

Make sure you understand the proof of part (ii), discussed in class (see the posted notes for an important point that we didn't emphasize).

Question 5. Review the basics of series and do Question 14.2. Please explain why the series converge or diverge. (You can use the comparison test, ratio test, and root test familiar from calculus; we will prove these tests in class.)

Question 6. Let $a_n, b_n > 0$ for all n. Suppose that $\sum a_n$ converges and the sequence (b_n) is bounded above. Prove that $\sum a_n b_n$ converges.