MAT 319
Homework 10
due Wednesday, April 13

Question 1. (a) In the proof of the Intermediate Value theorem (both in the book and in the posted notes) we assumed that $f(a) < f(b)$. Please write a detailed proof for the other case, $f(a) > f(b)$. You can follow the method from the book or the one from the notes.

(b) Assume the case $f(a) < f(b)$ as known. Give a quick proof of the Intermediate Value Theorem for the case $f(a) > f(b)$. Unlike part (a), DO NOT repeat all the steps of the proof in the other case. (For inspiration, compare questions 1 and 2 from Homework 9.)

Question 2. Please determine whether the following limits exist (and if they do, find them); prove all your statements carefully. (Saying things like "limit doesn't exist because the function oscillates" will get 0 points. This is an exercise in careful use of definitions. If referring to any theorems, make sure to use only those we actually proved.)

(a) $\lim_{x \to -\infty} x \cos x$

(b) $\lim_{x \to 0} x \cos x$

(c) $\lim_{x \to 0+} \frac{\cos x}{x}$

(d) $\lim_{x \to +\infty} \frac{\cos x}{x}$

Question 3. Give the definitions for

$$\lim_{x \to 2-} f(x) = -\infty$$

in terms of (1) sequences, (2) ϵ-δ approach, and show that your statements in (1) and (2) are equivalent. (I.e. if (1) holds, then (2) holds, and conversely.)

Please also do questions **20.14**, **20.16**, **20.18**, and **20.20**. Please justify everything: even for "obvious" statements like those in 20.14, you should make clear connections to definitions and all necessary theorems.