MAT 319 Introduction to Analysis

Homework 5
due Thursday, March 1

Please prove (or explain as appropriate) all your answers.

**Question 1.** In class, we showed by induction that $2^n > n$ for every positive integer $n$. Using this, show that
(a) the sequence $(2^n)$ diverges to $+\infty$
(b) the sequence $(\frac{1}{2^n})$ converges to 0.

**Question 2.** Prove that $\sqrt{5}$ is irrational. (Do not refer to Theorem 2.2 from the book that we didn’t cover. Give a proof from scratch.)

**Question 3.** Let $(x_n)$ be an increasing sequence.
(a) Prove that $(x_n)$ is bounded below.
(b) Suppose that $(x_n)$ is not bounded above. Prove that $x_n$ diverges to $+\infty$.

**Question 4.** (a) Prove that $1 \neq 0$.
Use the algebraic axioms A1-A4, M1-M4, DL on p. 13 as well as parts (i) and (ii) in Theorem 3.1. You can assume that the set $\mathbb{R}$ of real numbers has more than one element. Please give a proof from scratch, justify every step, and do not use any of the statements not listed above.
(b) Prove that $0 \leq 1$.
Use only the axioms A1-A4, M1-M4, DL, O1-O5 on p.13, and (i) and (ii) in Theorem 3.1 if needed. Justify every step.

Please also do questions 4.6 and 4.7(a) from the textbook.