Please prove (or explain as appropriate) all your answers.

**Question 1.** Find the following limits using limit laws (section 9). Explain carefully which theorems you used at each step.

(a) 
\[ x_n = \frac{2n - 1}{3n + 2} \]

(b) 
\[ x_n = \frac{7n^3 - n^2 + 1}{2n + 5n^3 - 3} \]

(c) 
\[ x_n = \frac{n}{n^4 + n^3 + n^2 - n + 1} \]

**Question 2.** (a) Suppose that \( (x_n) \) converges and \( (y_n) \) does not. Prove that \( (x_n + y_n) \) diverges.

(b) Suppose that both sequences \( (x_n) \) and \( (y_n) \) diverge. Is it possible that \( (x_n + y_n) \) converges? (Prove or give a counterexample.)

**Question 3.** Suppose that the sequence \( (x_n) \) converges to 4. Prove carefully that \( (\sqrt{x_n}) \) converges to 2. (Check also that for sufficiently large \( n \), \( \sqrt{x_n} \) is defined, i.e. \( x_n \) is non-negative.)

For your proof, the identity \((\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = a - b\) will be helpful.

**Question 4.** (a) Suppose that the sequence \( (x_n) \) converges to \( A \), the sequence \( (y_n) \) converges to \( B \), and \( x_n \leq y_n \) for every \( n \). Prove that \( A \leq B \).

(b) Suppose that, as in part (a), \( x_n \to A \), \( y_n \to B \), but now \( x_n < y_n \) for every \( n \). (The inequality is strict.) Give an example showing that the equality \( A = B \) is possible.