MAT 319 Introduction to Analysis

Homework 2

due Thursday, February 9

Please prove (or explain as appropriate) all your answers.

In questions 1-3, find limits and prove your answers carefully. Use the "formal" ϵ-definition of the limit. You may solve for N when it works, but you are also welcome to use rough estimates (in one of the questions, it will be necessary).

Question 1. Find the limit of the sequence

$$x_n = \frac{2n-1}{3n+2}$$

and prove your answer.

Question 2. Find the limit of the sequence

$$x_n = \frac{n^2}{n^2+1}$$

and prove your answer.

Question 3. Find the limit of the sequence

$$x_n = \frac{n}{n^4+n^3+n^2-n+1}$$

and prove your answer.

Question 4. Suppose that the sequence (x_n) converges to A. Working from the definition of the limit, prove that the sequence (y_n), where $y_n = 2x_n + 5$, converges to $2A + 5$.

In this question, you are required to argue from scratch. Do not refer to theorems in section 9.)

Question 5. Let (x_n) and (y_n) be two sequences, such that (x_n) converges to A, (y_n) converges to B, and $A \neq B$. Consider a new sequence where x_n's and y_n's alternate:

$$x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4, \ldots$$

Prove that this sequence *does not* converge.