Please prove (or explain as appropriate) all your answers.

In questions 1-3, find limits and prove your answers carefully. Use the “formal” $\epsilon$-definition of the limit. You may solve for $N$ when it works, but you are also welcome to use rough estimates (in one of the questions, it will be necessary).

**Question 1.** Find the limit of the sequence

$$x_n = \frac{2n - 1}{3n + 2}$$

and prove your answer.

**Question 2.** Find the limit of the sequence

$$x_n = \frac{n^2}{n^2 + 1}$$

and prove your answer.

**Question 3.** Find the limit of the sequence

$$x_n = \frac{n}{n^4 + n^3 + n^2 - n + 1}$$

and prove your answer.

**Question 4.** Suppose that the sequence $(x_n)$ converges to $A$. Working from the definition of the limit, prove that the sequence $(y_n)$, where $y_n = 2x_n + 5$, converges to $2A + 5$.

*In this question, you are required to argue from scratch. Do not refer to theorems in section 9.*

**Question 5.** Let $(x_n)$ and $(y_n)$ be two sequences, such that $(x_n)$ converges to $A$, $(y_n)$ converges to $B$, and $A \neq B$. Consider a new sequence where $x_n$’s and $y_n$’s alternate:

$$x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4, \ldots$$

Prove that this sequence *does not* converge.