MAT 319 Introduction to Analysis

Homework 1

due Thursday, February 2

Please prove (or explain as appropriate) all your answers.

Question 1. On the real line, sketch ϵ-neighborhoods of $A = -1$ for $\epsilon = 3$, $\epsilon = 1$, $\epsilon = 1/2$.

For each of these neighborhoods, determine whether it contains points $x = 2$, $y = -2$, $z = 1$, $t = -1/2$, $v = 1/2$. (Plot these points to explain and illustrate your answer.)

Question 2. Consider the sequence (s_n), where

$$s_n = \begin{cases} n & \text{if } n \le 100 \\ 2 & \text{if } n > 100. \end{cases}$$

Does this sequence converge? Prove your answer.

Question 3. Determine whether the following sequences converge. If so, find the limits. Prove your answers.

(a) the sequence (x_n), where $x_n = \frac{1}{n+17}$.

(b) the sequence (y_n), where

$$y_n = \begin{cases} 0 & \text{if } n = 17 \\ \frac{1}{n-17} & \text{otherwise} \end{cases}$$

Question 4. Suppose that the sequence (x_n) converges to 0. Prove that the sequence

$$x_1, 0, x_2, 0, x_3, 0, x_4, 0, \dots,$$

constructed by alternating the terms of the sequence (x_n) with zeroes, also has limit 0.

Question 5. Let (x_n) and (y_n) be two sequences converging to 0. Consider a new sequence where x_n's and y_n's alternate:

$$x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4, \dots$$

Prove that this sequence converges to 0.

Use the neighborhoods-and-tails definition of a limit. In most questions here, it is not only more visual but also easier.