Please prove (or explain as appropriate) all your answers.

**Question 1.** On the real line, sketch \( \epsilon \)-neighborhoods of \( A = -1 \) for \( \epsilon = 3, \epsilon = 1, \epsilon = 1/2 \).

For each of these neighborhoods, determine whether it contains points \( x = 2, y = -2, z = 1, t = -1/2, v = 1/2 \). (Plot these points to explain and illustrate your answer.)

**Question 2.** Consider the sequence \((s_n)\), where
\[
s_n = \begin{cases} 
  n & \text{if } n \leq 100 \\
  2 & \text{if } n > 100.
\end{cases}
\]

Does this sequence converge? Prove your answer.

**Question 3.** Determine whether the following sequences converge. If so, find the limits. Prove your answers.
(a) the sequence \((x_n)\), where \( x_n = \frac{1}{n+17} \).
(b) the sequence \((y_n)\), where
\[
y_n = \begin{cases} 
  0 & \text{if } n = 17 \\
  \frac{1}{n-17} & \text{otherwise}
\end{cases}
\]

**Question 4.** Suppose that the sequence \((x_n)\) converges to 0. Prove that the sequence
\[x_1, 0, x_2, 0, x_3, 0, x_4, 0, \ldots,\]
constructed by alternating the terms of the sequence \((x_n)\) with zeroes, also has limit 0.

**Question 5.** Let \((x_n)\) and \((y_n)\) be two sequences converging to 0. Consider a new sequence where \(x_n\’s\) and \(y_n\’s\) alternate:
\[x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4, \ldots\]

Prove that this sequence converges to 0.

Use the neighborhoods-and-tails definition of a limit. In most questions here, it is not only more visual but also easier.