Curves Encomplexed

Oleg Viro

October 31, 2006

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Encomplex

Many objects studied in geometry are defined in real coordinates by equations.

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Encomplex

Many objects studied in geometry are defined in real coordinates by equations.
Often, the equations make sense even for complex values of coordinates

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Many objects studied in geometry are defined in real coordinates by equations.
Often, the equations make sense even for complex values of coordinates, and define the corresponding objects in the complex space.

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Encomplex

Many objects studied in geometry are defined in real coordinates by equations.
Often, the equations make sense even for complex values of coordinates, and define the corresponding objects in the complex space.

The new complex objects are even nicer, although they are less visual.

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Many objects studied in geometry are defined in real coordinates by equations.
Often, the equations make sense even for complex values of coordinates, and define the corresponding objects in the complex space.
The new complex objects are even nicer.
When possible, mathematicians tend to switch to them.

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Often, the equations make sense even for complex values of coordinates, and define the corresponding objects in the complex space.

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When possible, mathematicians tend to switch to them.
This is how algebraic geometry became complex.

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Many objects studied in geometry are defined in real coordinates by equations.
Often, the equations make sense even for complex values of coordinates, and define the corresponding objects in the complex space.
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Real objects are replaced by their complex counter-parts, complexifications.

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I will call this *to encomplex*

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Real objects are replaced by their complex counter-parts, complexifications. This is called *complexification*.

Another option is to consider the original objects embedded into its complexification. More difficult, but nonetheless rewarding!

I will call this *to encomplex* and try to show its difficulties and advantages on a simple material of curves.

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A real plane curve is a generically immersed circle

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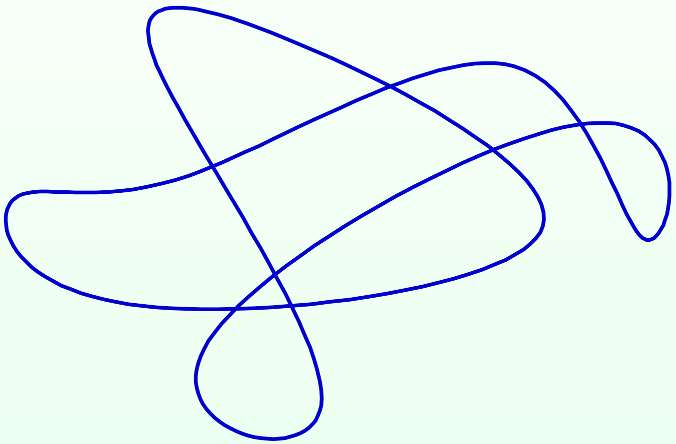
Curves

A real plane curve is a generically immersed circle,

immersion $S^1 \looparrowright \mathbb{R}^2$



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Curves

A real plane curve is a generically immersed circle, immersion $S^1 \looparrowright \mathbb{R}^2$, belongs to Differential Geometry,

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A real plane curve is a generically immersed circle, immersion $S^1 \looparrowright \mathbb{R}^2$, belongs to Differential Geometry, presumably has no complexification.

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Curves

A real plane curve is a generically immersed circle, immersion $S^1 \looparrowright \mathbb{R}^2$, belongs to Differential Geometry, presumably has no complexification.

There are results on generic plane curves with a global topological flavor.

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Curves

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There are results on generic plane curves with a global topological flavor.

One of the most classical of them is the Whitney classification of curves up to regular homotopy.

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Curves

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One of the most classical of them is the Whitney classification of curves up to regular homotopy.

The next masterpiece is Arnold's theory on three first order invariants of generic plane curves.

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Curves

A real plane curve is a generically immersed circle, immersion $S^1 \looparrowright \mathbb{R}^2$, belongs to Differential Geometry, presumably has no complexification.

There are results on generic plane curves with a global topological flavor.

One of the most classical of them is the Whitney classification of curves up to regular homotopy.

The next masterpiece is Arnold's theory on three first order invariants of generic plane curves.

I am going to encomplex them in this talk.

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Require algebraicity

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A generic immersion $S^1 \looparrowright \mathbb{R}^2$ is not assumed to have a complexification.
Require algebraicity

that is assume that the curve-image is defined by a polynomial equation

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Require algebraicity, and you get complex points.

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- Geometry hidden in complexification:

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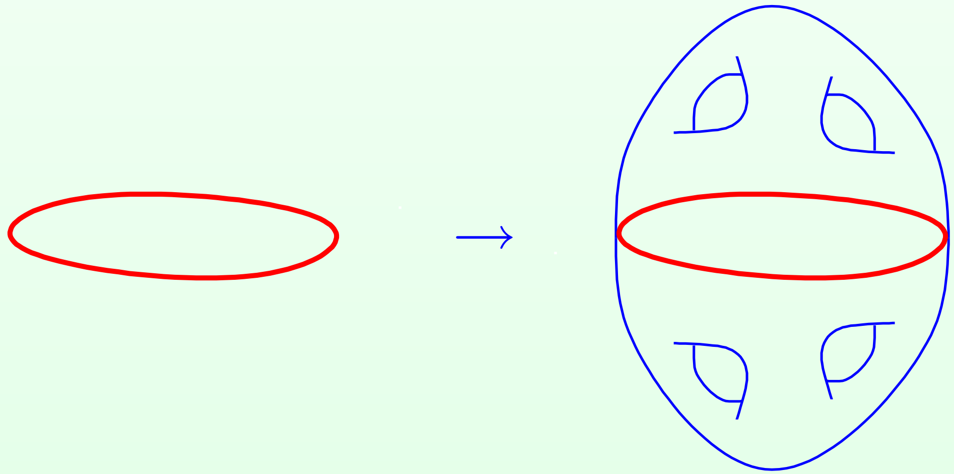
A generic immersion $S^1 \looparrowright \mathbb{R}^2$ is not assumed to have a complexification.

Require algebraicity, and you get complex points.

What's in a complex view?

- Geometry hidden in complexification: genus,



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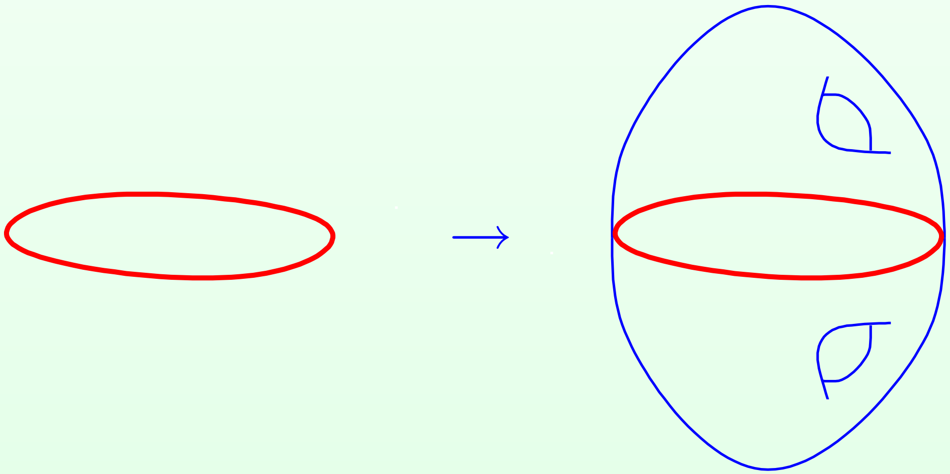

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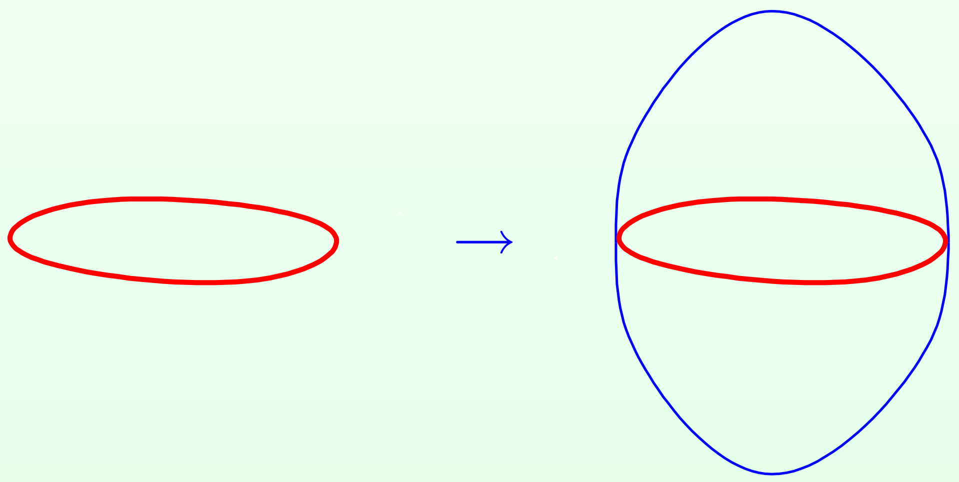

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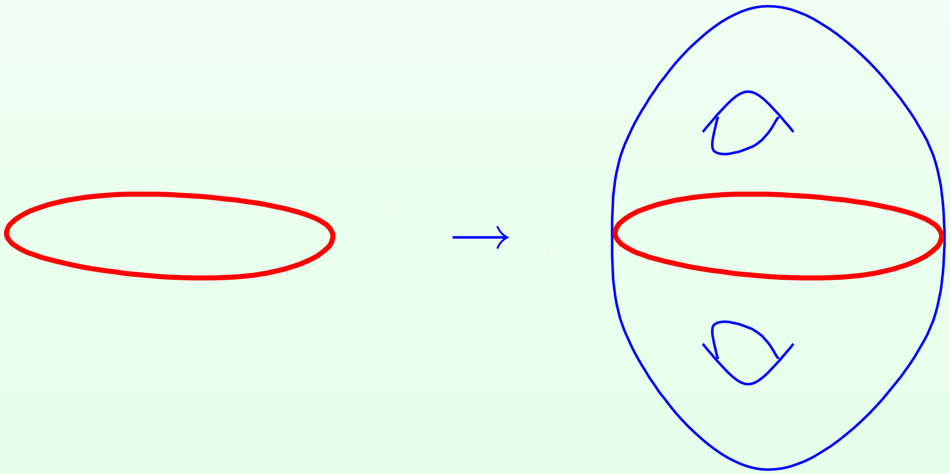

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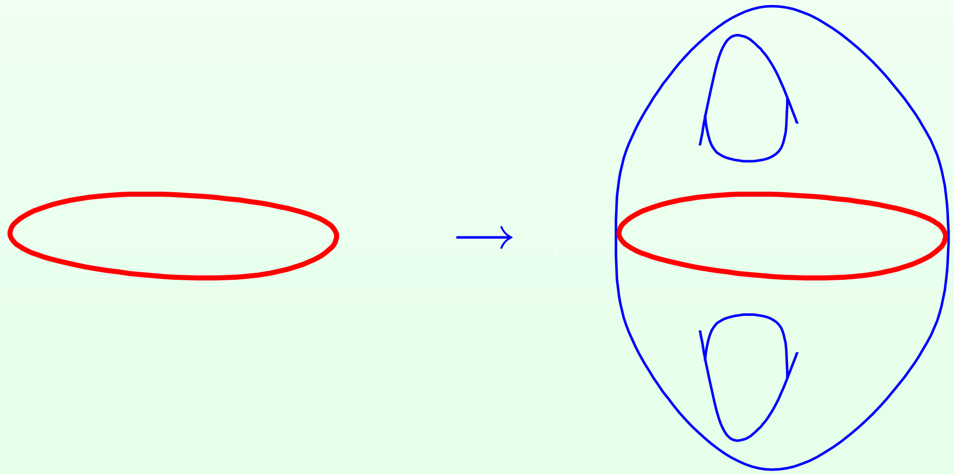

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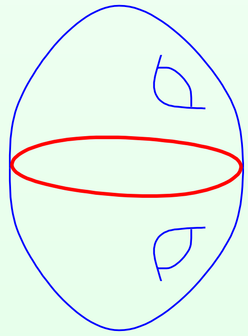
Require algebraicity, and you get complex points.

What's in a complex view?

● Geometry hidden in complexification: genus, moduli, type (of complex conjugation).



Type I: the set of real points divides the set of complex points into two connected components.

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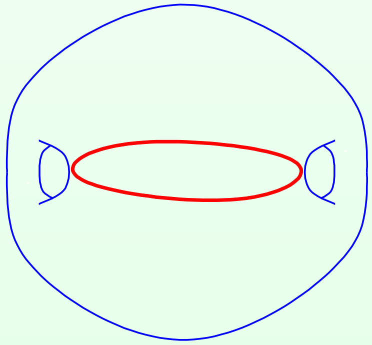
A generic immersion $S^1 \looparrowright \mathbb{R}^2$ is not assumed to have a complexification.

Require algebraicity, and you get complex points.

What's in a complex view?

● Geometry hidden in complexification: genus, moduli, type.



Type II: the set of real points does not divide the set of complex points.

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A generic immersion $S^1 \looparrowright \mathbb{R}^2$ is not assumed to have a complexification.

Require algebraicity, and you get complex points.

What's in a complex view?

● Geometry hidden in complexification: genus, moduli, type.

● Interaction between real and complex.

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● Geometry hidden in complexification: genus, moduli, type.

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Whitney number is related to complex asymptotes.

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Require algebraicity, and you get complex points.

What's in a complex view?

- Geometry hidden in complexification: genus, moduli, type.
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Arnold's invariant J_- is related to
the number of imaginary intersection points of complex halves.

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- Geometry hidden in complexification: genus, moduli, type.
- Interaction between real and complex.
- Results on real curves inspired by results on curves with complexification.

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Formula for J_-:

$$J_-(C) = 1 - \int_{\mathbb{R}^2 \setminus \widetilde{C}} (ind_{\widetilde{C}}(x))^2 \, d\chi(x).$$

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● Geometry hidden in complexification: genus, moduli, type.

● Interaction between real and complex.

● Results on real curves inspired by results on curves with complexification.

● A world parallel to Real Geometry.

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Arnold's strangeness of rational real algebraic curves.

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The simplest complexification of curves are rational curves: genus zero, no moduli, polynomial parametrization.

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For an oriented smooth closed immersed curve C on plane

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For an oriented smooth closed immersed curve C on plane
$w(C)$, *Whitney number*

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For an oriented smooth closed immersed curve C on plane

$w(C)$, *Whitney number*

$=$ rotation number of the velocity vector

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For an oriented smooth closed immersed curve C on plane $w(C)$, *Whitney number*
$=$ rotation number of the velocity vector
$=$ degree of the Gauss map $C \to S^1$.

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Whitney number

For an oriented smooth closed immersed curve C on plane
$w(C)$, *Whitney number*
$=$ rotation number of the velocity vector
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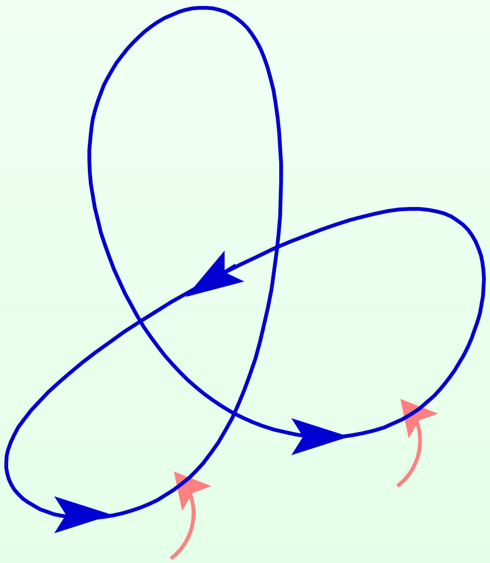
Example.



$w(C) = +2$

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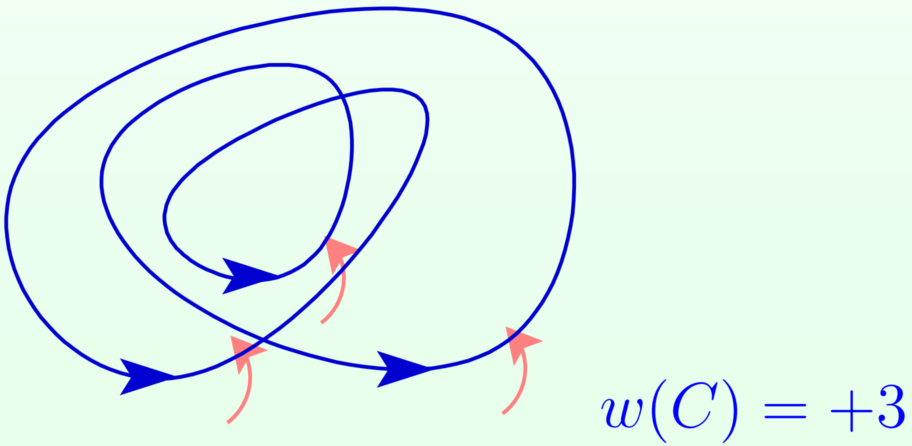

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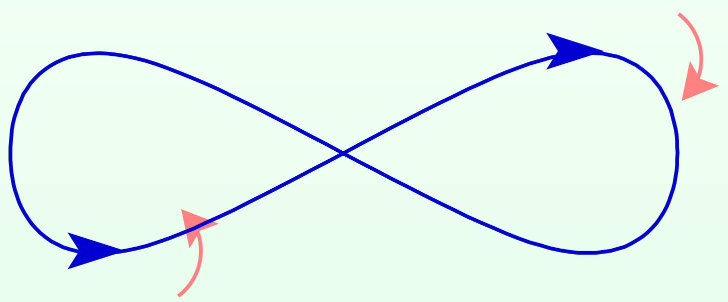

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Whitney number

For an oriented smooth closed immersed curve C on plane

$w(C)$, *Whitney number*

$=$ rotation number of the velocity vector

$=$ degree of the Gauss map $C \to S^1$.

Whitney Theorem.

$w(C)$ *determines* $C : S^1 \looparrowright \mathbb{R}^2$ *up to regular homotopy.*

choice of curves

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Consider irreducible plane affine real algebraic curves A such that

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Consider irreducible plane affine real algebraic curves A such that:

- $\mathbb{R}A$ is compact,
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choice of curves

Consider irreducible plane affine real algebraic curves A such that:

- $\mathbb{R}A$ is compact, real branches don't go to infinity!
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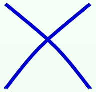
Consider irreducible plane affine real algebraic curves A such that:

- $\mathbb{R}A$ is compact,
- all real singularities are  's,
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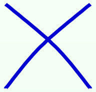
- all real singularities are 's, $\mathbb{R}A$ generically immersed
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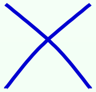
- all real singularities are 's,
- $\mathbb{R}A$ is zero homologous modulo 2 in $\mathbb{C}A \subset \mathbb{C}P^2$

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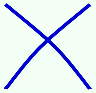
Encomplexing J_-

Consider irreducible plane affine real algebraic curves A such that:

- $\mathbb{R}A$ is compact,
- all real singularities are  's,
- $\mathbb{R}A$ is zero homologous modulo 2 in $\mathbb{C}A \subset \mathbb{C}P^2$

to be naturally oriented

choice of curves

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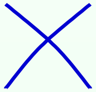
- all real singularities are  's,
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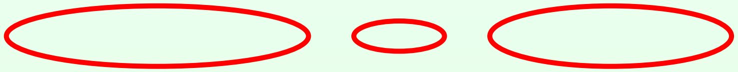

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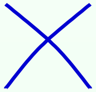
- all real singularities are  's,
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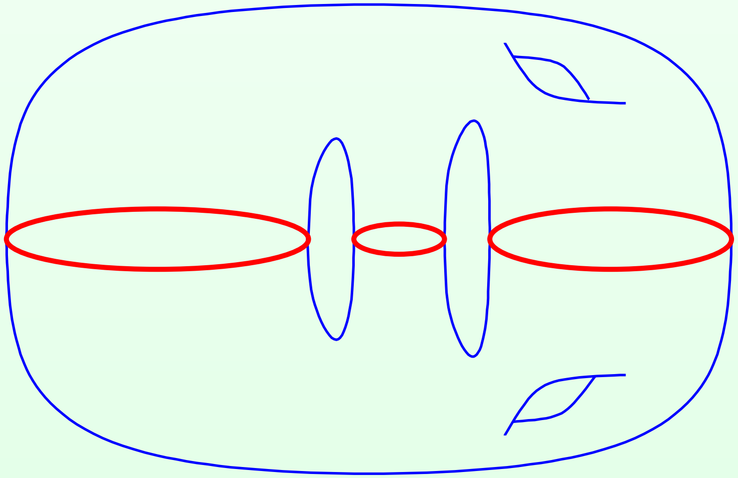

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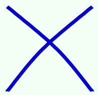
- all real singularities are 's,
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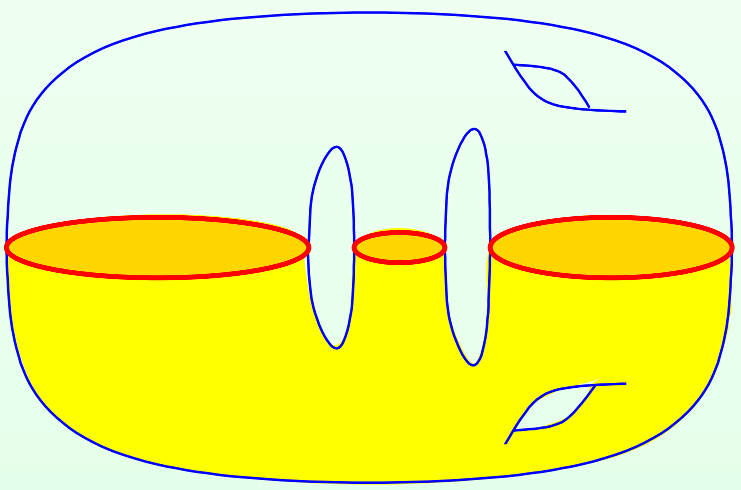

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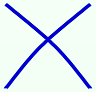
- all real singularities are  's,
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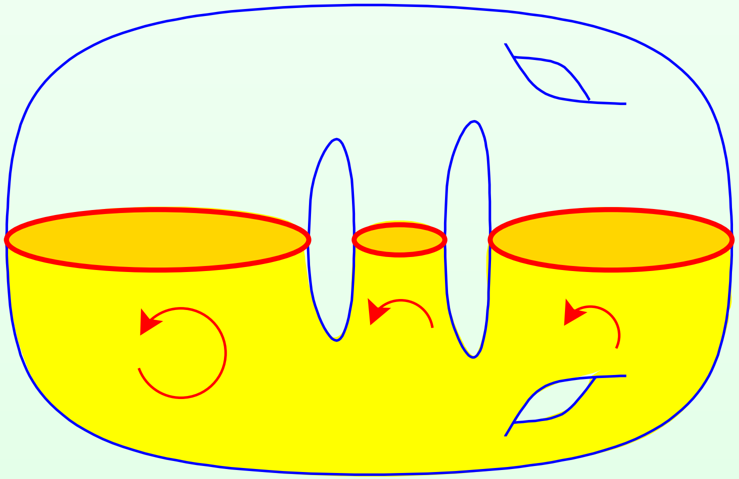

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Consider irreducible plane affine real algebraic curves A such that:

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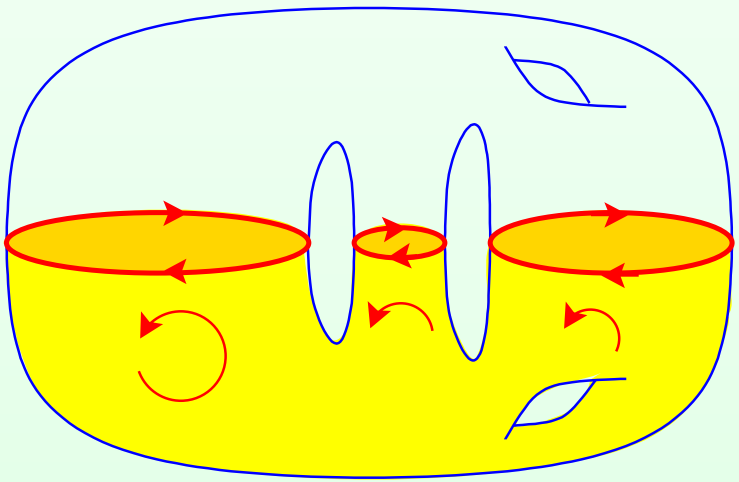
to be naturally oriented



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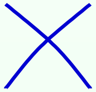
- all real singularities are  's,
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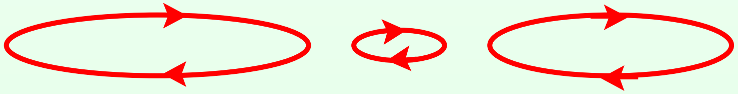

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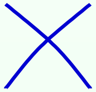
- all real singularities are  's,
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If $\mathbb{R}A$ is zero homologous in $\mathbb{C}A$ then A is said to be of *type I*.

(Felix Klein)

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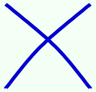
- all real singularities are  's,
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(Felix Klein)

Any real rational curve with infinite $\mathbb{R}A$ is of type I.

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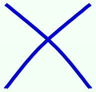
- all real singularities are  's,
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If $\mathbb{R}A$ is zero homologous in $\mathbb{C}A$ then A is said to be of *type I*.

(Felix Klein)

Any normal A of genus g such that $\mathbb{R}A$ has $g+1$ components is of type I.

choice of curves

Consider irreducible plane affine real algebraic curves A such that:

- $\mathbb{R}A$ is compact,
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If $\mathbb{R}A$ is zero homologous in $\mathbb{C}A$ then A is said to be of *type I*.

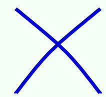
(Felix Klein)

Type I implies:

$$b_0(\mathbb{R}\text{ normalized } A) \equiv \text{genus}(A) + 1 \mod 2.$$

choice of curves

Consider irreducible plane affine real algebraic curves A such that:

- $\mathbb{R}A$ is compact,
- all real singularities are  's,
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(Felix Klein)

The orientation of $\mathbb{R}A$ induced from $\mathbb{C}A_+ \subset \mathbb{C}A$ with $\partial\mathbb{C}A_+ = \mathbb{R}A$ is called a *complex orientation*. (V.A.Rokhlin)

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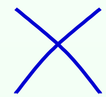
- all real singularities are  's,
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If $\mathbb{R}A$ is zero homologous in $\mathbb{C}A$ then A is said to be of *type I*.

(Felix Klein)

The orientation of $\mathbb{R}A$ induced from $\mathbb{C}A_+ \subset \mathbb{C}A$ with $\partial \mathbb{C}A_+ = \mathbb{R}A$ is called a *complex orientation*. (V.A.Rokhlin)
Denote $\mathbb{R}A$ equipped with the orientation induced from $\mathbb{C}A_+ \subset \mathbb{C}A$ by $\mathbb{R}A_+$.

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complex line at infinity

$$\mathbb{C}P^1_\infty = \mathbb{C}P^2 \smallsetminus \mathbb{C}^2,$$

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complex line at infinity

$$\mathbb{C}P^1_\infty = \mathbb{C}P^2 \smallsetminus \mathbb{C}^2, \quad \mathbb{R}P^1_\infty = \mathbb{R}P^2 \smallsetminus \mathbb{R}^2$$

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complex line at infinity

$\mathbb{C}P^1_\infty = \mathbb{C}P^2 \smallsetminus \mathbb{C}^2$, $\mathbb{R}P^1_\infty = \mathbb{R}P^2 \smallsetminus \mathbb{R}^2$

Denote $\mathbb{R}P^1_\infty$ equipped with the orientation induced by the standard orientation of $\mathbb{R}^2$ by $\mathbb{R}P^1_{\infty+}$.

say, counter-clockwise orientation of $\mathbb{R}^2$.

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complex line at infinity

$$\mathbb{C}P^1_\infty = \mathbb{C}P^2 \smallsetminus \mathbb{C}^2, \quad \mathbb{R}P^1_\infty = \mathbb{R}P^2 \smallsetminus \mathbb{R}^2$$

Denote $\mathbb{R}P^1_\infty$ equipped with the orientation induced by the standard orientation of $\mathbb{R}^2$ by $\mathbb{R}P^1_{\infty+}$.

Denote by $\mathbb{C}P^1_{\infty+}$ the hemisphere of $\mathbb{C}P^1_\infty$ with $\partial\mathbb{C}P^1_{\infty+} = \mathbb{R}P^1_{\infty+}$.

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$$\mathbb{C}P^1_\infty = \mathbb{C}P^2 \setminus \mathbb{C}^2, \quad \mathbb{R}P^1_\infty = \mathbb{R}P^2 \setminus \mathbb{R}^2$$

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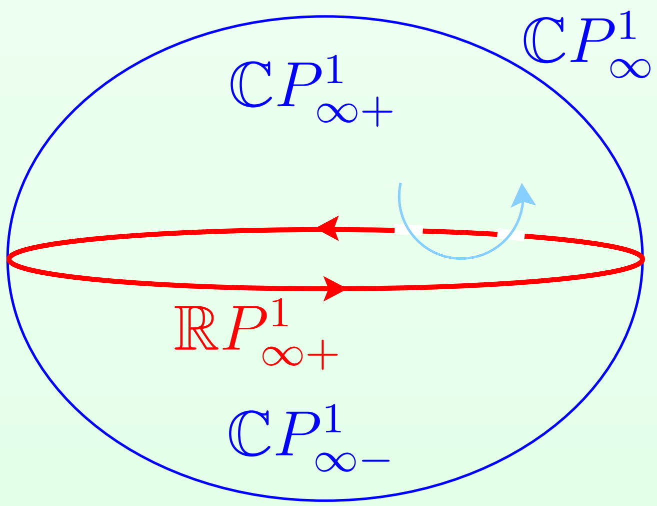

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Theorem 1

Let A *be a plane affine real algebraic curve of type I,*

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Theorem 1

Let A *be a plane affine real algebraic curve of type I, such that*

- $\mathbb{R}A$ *is compact,*

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Let A *be a plane affine real algebraic curve of type I, such that*

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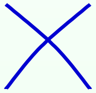
- *all real singularities are*  *'s.*

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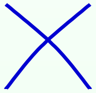
- *all real singularities are*  *'s.*

Then $w(\mathbb{R}A_+) = \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty+} - \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty-}$.

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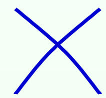
- *all real singularities are*  *'s.*

Then $w(\mathbb{R}A_+) = \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty+} - \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty-}$.

Corollary. $|w(\mathbb{R}A)| \leq \frac{1}{2} \deg A.$

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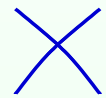
- *all real singularities are*  *'s.*

Then $w(\mathbb{R}A_+) = \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty+} - \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty-}$ *.*

Corollary. $|w(\mathbb{R}A)| \leq \frac{1}{2} \deg A$.

Indeed, $|w(\mathbb{R}A)| = |\mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty+} - \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty-}|$

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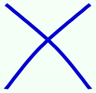
- *all real singularities are*  *'s.*

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Corollary. $|w(\mathbb{R}A)| \le \frac{1}{2}\deg A$.

Indeed,
$$|w(\mathbb{R}A)| = |\mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty +} - \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty -}| \le |\mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty +} + \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty -}|$$

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Indeed,
$$\begin{aligned} |w(\mathbb{R}A)| &= |\mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty +} - \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty -}| \\ &\leq |\mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty +} + \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty -}| \\ &= |\mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty}| \end{aligned}$$

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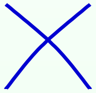
- *all real singularities are*  *'s.*

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in terms of asymptotes

If $\mathbb{C}A \pitchfork \mathbb{C}P^1_\infty$, then each point of $\mathbb{C}A \cap \mathbb{C}P^1_\infty$ corresponds to an asymptote of $\mathbb{C}A \cap \mathbb{C}^2$.

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Asymptotes are imaginary.

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Asymptotes are imaginary.

An imaginary line disjoint with $\mathbb{R}P^1_\infty$ meets either $\mathbb{C}P^1_{\infty+}$ or $\mathbb{C}P^1_{\infty-}$.

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in terms of asymptotes

If $\mathbb{C}A \pitchfork \mathbb{C}P^1_\infty$, then each point of $\mathbb{C}A \cap \mathbb{C}P^1_\infty$ corresponds to an asymptote of $\mathbb{C}A \cap \mathbb{C}^2$.

Asymptotes are imaginary.

An imaginary line disjoint with $\mathbb{R}P^1_\infty$ meets either $\mathbb{C}P^1_{\infty+}$ or $\mathbb{C}P^1_{\infty-}$.

Theorem 1 says:

$w(\mathbb{R}A_+)$ *equals the difference between the numbers of the asymptotes of* $\mathbb{C}A_+ \cap \mathbb{C}^2$ *of these two sorts.*

near a kiss

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Lemma **(Imaginary intersection after a real kiss)**

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Lemma (Imaginary intersection after a real kiss)
Let A and B be curves of type I,

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Lemma (Imaginary intersection after a real kiss)
Let A and B be curves of type I,
with $\mathbb{R}A_+$ and $\mathbb{R}B_+$ almost kissing each other
near a point p.

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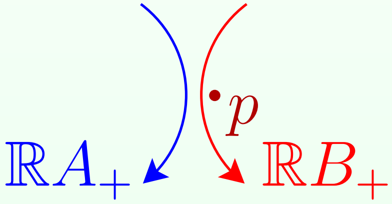

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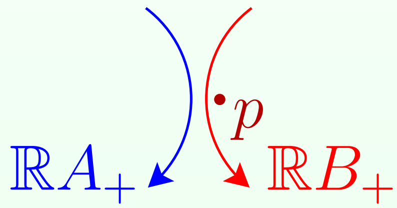

Then $\mathbb{C}A_+$ meets $\mathbb{C}B_+$ at an imaginary point near p

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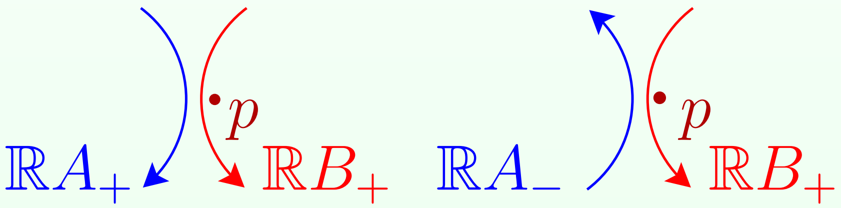

Then $\mathbb{C}A_+$ meets $\mathbb{C}B_+$ at an imaginary point near p,
while $\mathbb{C}A_-$ does not.

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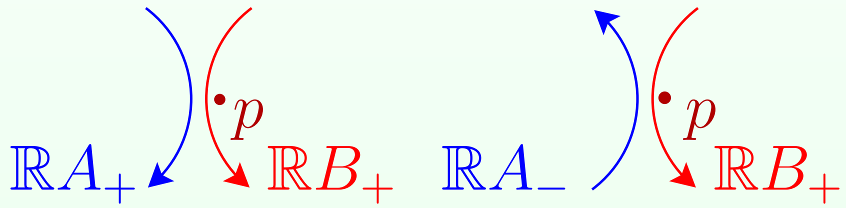

Then $\mathbb{C}A_+$ meets $\mathbb{C}B_+$ at an imaginary point near p,
while $\mathbb{C}A_-$ does not.

Proof. Look at the scene complexly from the left hand side.

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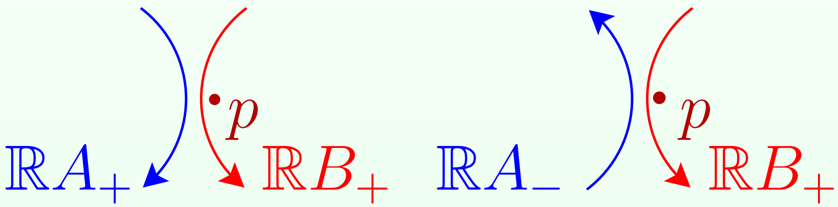

Then $\mathbb{C}A_+$ meets $\mathbb{C}B_+$ at an imaginary point near p,
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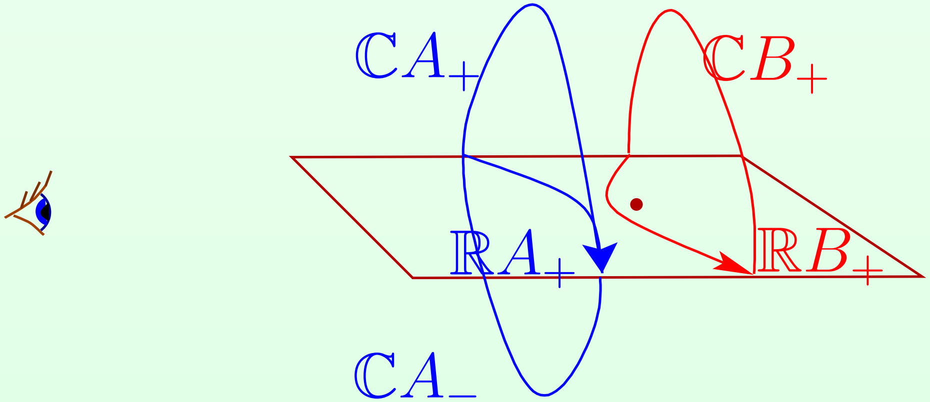
Proof. Look at the scene complexly from the left hand side.



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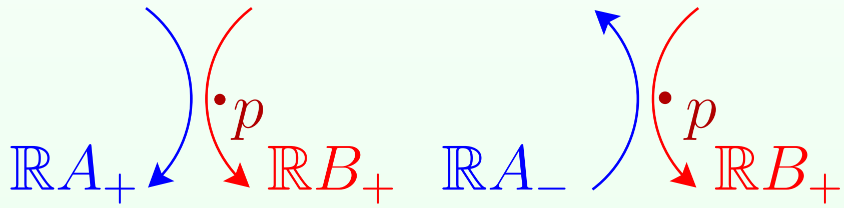
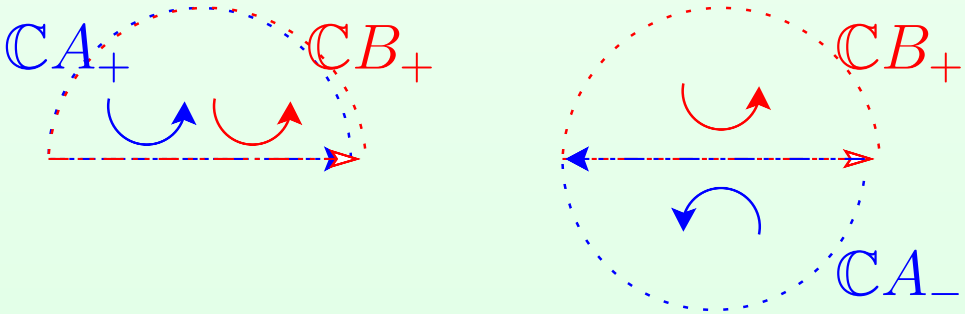


Then $\mathbb{C}A_+$ meets $\mathbb{C}B_+$ at an imaginary point near p,
while $\mathbb{C}A_-$ does not.

Proof. Look at the scene complexly from the left hand side.

Pictures:



proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and

proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p

proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p

(In $\mathbb{R}^2$ rotation around a point at infinity is a translation.)

proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p counting changes of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$.

proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p counting changes of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$.

We consider only imaginary intersection points.

proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p counting changes of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$.

We consider only imaginary intersection points.

(Although we start with $\mathbb{R}L = \mathbb{R}P^1_\infty$ and $\mathbb{R}P^1_\infty \cap \mathbb{R}A = \varnothing$, $\mathbb{R}L$ sweeps the whole $\mathbb{R}A$ while moving).

proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p counting changes of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$.
$\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$ changes, when $\mathbb{R}L$ gets tangent to $\mathbb{R}A$.

proof of Theorem 1

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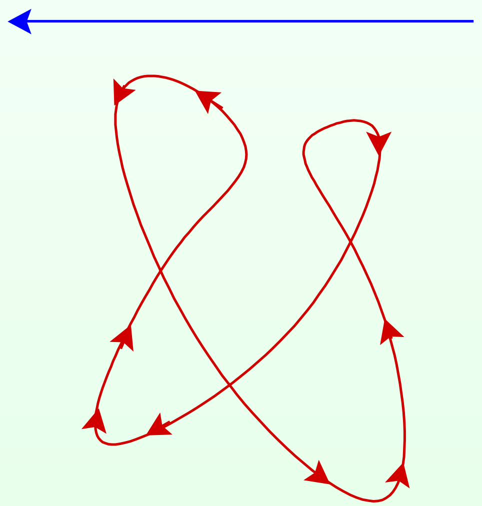
$\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$ changes, when $\mathbb{R}L$ gets tangent to $\mathbb{R}A$.
At these moments, evaluate also local degree $ldeg$ of Gauss map $\mathbb{R}A \to S^1$.

proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p counting changes of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$.
$\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$ changes, when $\mathbb{R}L$ gets tangent to $\mathbb{R}A$.
At these moments, evaluate also local degree $ldeg$ of Gauss map $\mathbb{R}A \to S^1$.

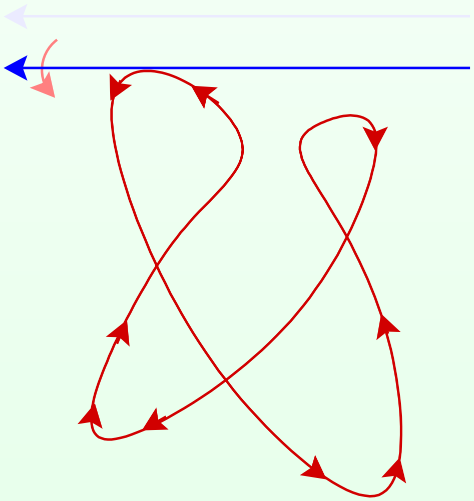
Consider, for example, the following curve:



proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p counting changes of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$.
$\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$ changes, when $\mathbb{R}L$ gets tangent to $\mathbb{R}A$.
At these moments, evaluate also local degree $ldeg$ of Gauss map $\mathbb{R}A \to S^1$.

$\Delta(\mathbb{C}A_+ \circ \mathbb{C}L_+) = -1$,
$\Delta(\mathbb{C}A_+ \circ \mathbb{C}L_-) = 0$,
$\Delta(\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-) = -1$
$ldeg = +1$



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Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p counting changes of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$.
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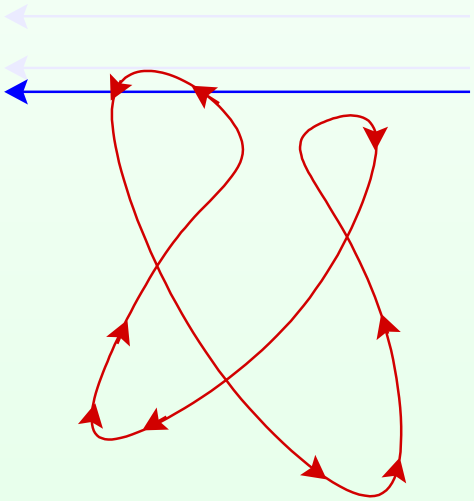

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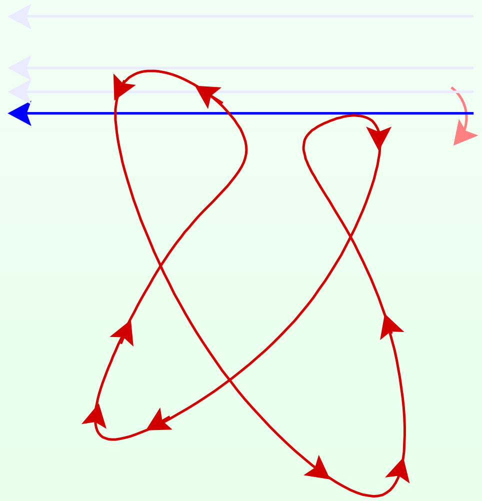
At these moments, evaluate also local degree $ldeg$ of Gauss map $\mathbb{R}A \to S^1$.

$\Delta(\mathbb{C}A_+ \circ \mathbb{C}L_+) = 0$,
$\Delta(\mathbb{C}A_+ \circ \mathbb{C}L_-) = -1$,
$\Delta(\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-) = +1$
$ldeg = -1$



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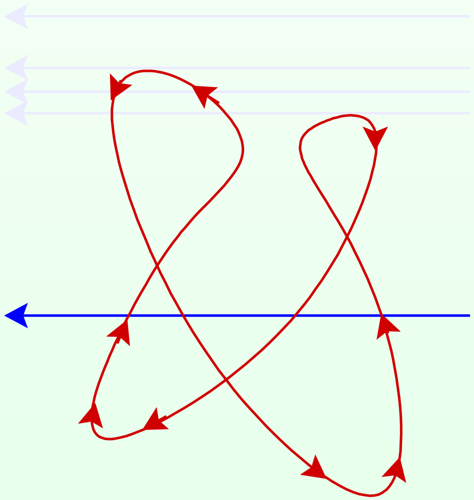

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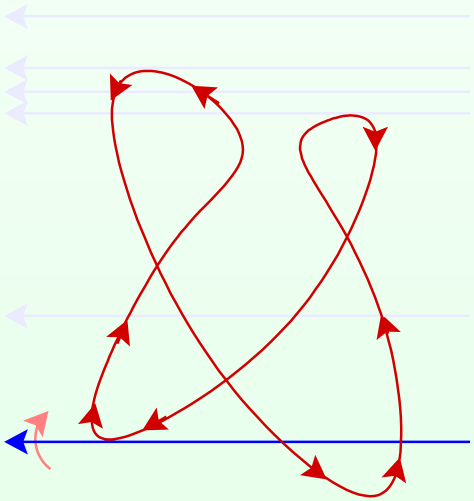

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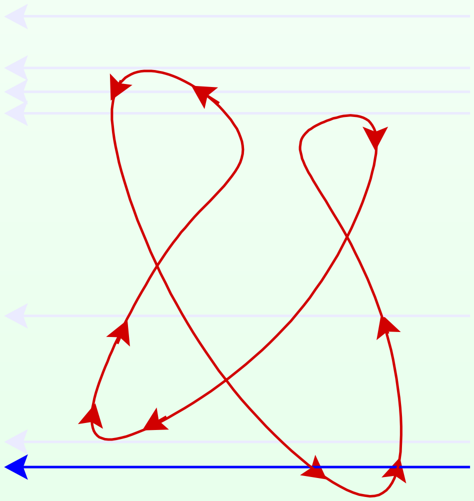

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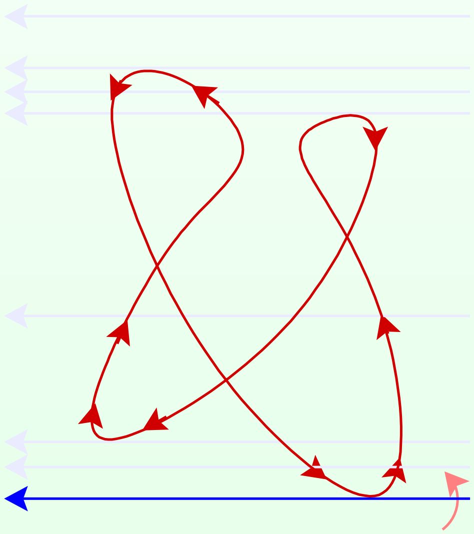

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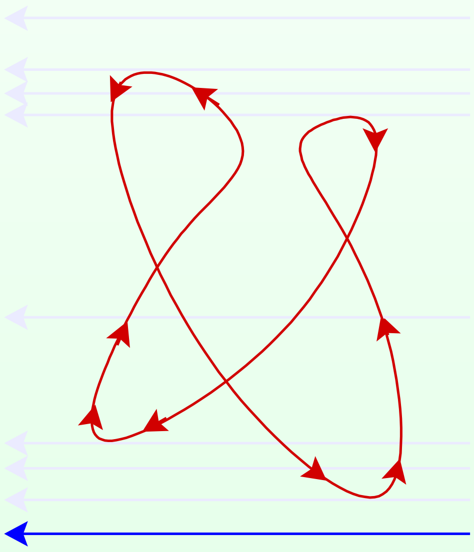

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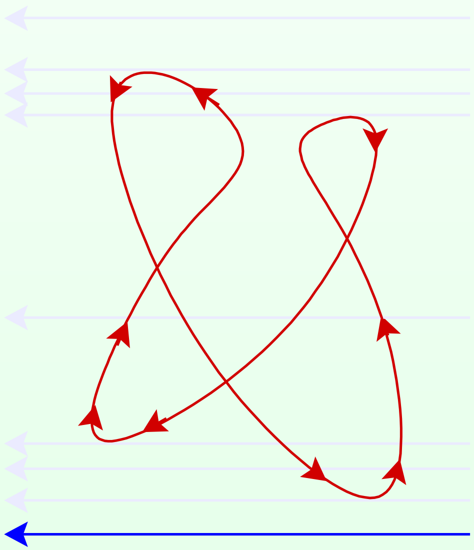

At each of the moments, $\Delta(\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-) = -ldeg$.

proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p counting changes of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$.
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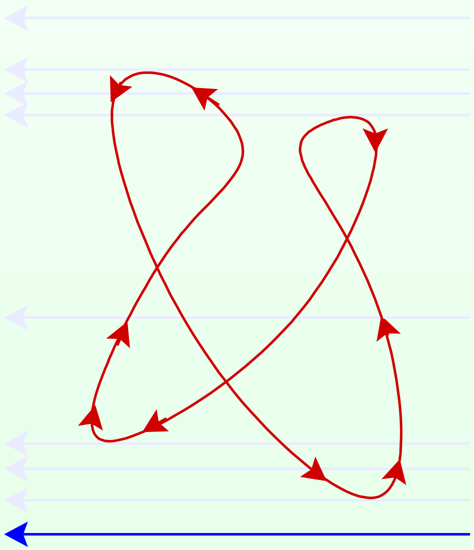

At each of the moments, $\Delta(\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-) = -ldeg$.
The full change of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$ is $-2w(\mathbb{R}A)$, since we have summed up $-ldeg$ over the preimages of 2 points.

proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p counting changes of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$.
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The full change of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$ is $-2w(\mathbb{R}A)$.

proof of Theorem 1

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At these moments, evaluate also local degree $ldeg$ of Gauss map $\mathbb{R}A \to S^1$.

The full change of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$ is $-2w(\mathbb{R}A)$.

On the other hand, the full change is

$$-2(\mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty +} - \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty -})$$

proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p counting changes of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$.
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On the other hand, the full change is

$$-2(\mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty+} - \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty-})$$

Indeed, we have turned $\mathbb{R}P^1_\infty$ by π,

proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p counting changes of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$.
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At these moments, evaluate also local degree $ldeg$ of Gauss map $\mathbb{R}A \to S^1$.

The full change of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$ is $-2w(\mathbb{R}A)$.

On the other hand, the full change is

$$-2(\mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty+} - \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty-})$$

Indeed, we have turned $\mathbb{R}P^1_\infty$ by π,
its orientation has reversed,

proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p counting changes of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$.
$\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$ changes, when $\mathbb{R}L$ gets tangent to $\mathbb{R}A$.
At these moments, evaluate also local degree $ldeg$ of Gauss map $\mathbb{R}A \to S^1$.

The full change of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$ is $-2w(\mathbb{R}A)$.

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Indeed, we have turned $\mathbb{R}P^1_\infty$ by π,
its orientation has reversed, and $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$ evolved from $\mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty+} - \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty-}$ to
$\mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty-} - \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty+} = -(\mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty+} - \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty-})$.

proof of Theorem 1

Choose a generic point p on $\mathbb{R}P^1_\infty$ and rotate oriented real line L around p counting changes of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$.
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At these moments, evaluate also local degree $ldeg$ of Gauss map $\mathbb{R}A \to S^1$.

The full change of $\mathbb{C}A_+ \circ \mathbb{C}L_+ - \mathbb{C}A_+ \circ \mathbb{C}L_-$ is $-2w(\mathbb{R}A)$.

On the other hand, the full change is

$$-2(\mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty+} - \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty-})$$

Thus,

$$w(\mathbb{R}A) = \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty+} - \mathbb{C}A_+ \circ \mathbb{C}P^1_{\infty-}$$

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The expression provided by Theorem 1 for Whitney number seems to be more stable than the Whitney number itself: not $\mathcal{C}^1$, but $\mathcal{C}^0$. $w(C)$ changes by 1, when C moves like that:

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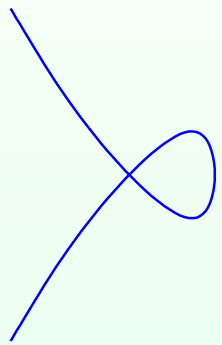

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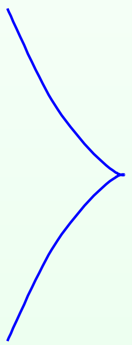

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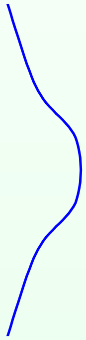

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The expression provided by Theorem 1 for Whitney number seems to be more stable than the Whitney number itself: not $\mathcal{C}^1$, but $\mathcal{C}^0$. $w(C)$ changes by 1, when C moves like that:

However, this move is impossible for algebraic curves of type I.

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However, this move is impossible for algebraic curves of type I. A real double point cannot disappear by becoming imaginary alone. It takes two, to become imaginary!

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However, this move is impossible for algebraic curves of type I. A real double point cannot disappear by becoming imaginary alone. Instead, it can turn into a double point isolated on $\mathbb{R}A$ with imaginary complex conjugate branches.

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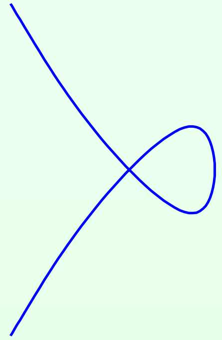

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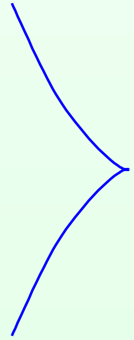

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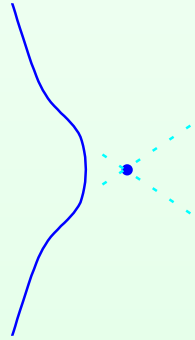

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The expression provided by Theorem 1 for Whitney number seems to be more stable than the Whitney number itself: not $\mathcal{C}^1$, but $\mathcal{C}^0$. $w(C)$ changes by 1, when C moves like that:

However, this move is impossible for algebraic curves of type I. A real double point cannot disappear by becoming imaginary alone. Instead, it can turn into a double point isolated on $\mathbb{R}A$ with imaginary complex conjugate branches. A double real point with imaginary branches is not allowed in Theorem 1.

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However, this move is impossible for algebraic curves of type I. A real double point cannot disappear by becoming imaginary alone. Instead, it can turn into a double point isolated on $\mathbb{R}A$ with imaginary complex conjugate branches. A double real point with imaginary branches is not allowed in Theorem 1. Allow such points,

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However, this move is impossible for algebraic curves of type I. A real double point cannot disappear by becoming imaginary alone. Instead, it can turn into a double point isolated on $\mathbb{R}A$ with imaginary complex conjugate branches. A double real point with imaginary branches is not allowed in Theorem 1. Allow such points, but take into account their contribution to $w(\mathbb{R}A_+)$. Only one of the branches passing through it, belongs to $\mathbb{C}A_+$. Its intersection number with $\mathbb{R}^2$ is to be added to $w(\mathbb{R}A_+)$.

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However, this move is impossible for algebraic curves of type I. A real double point cannot disappear by becoming imaginary alone. Instead, it can turn into a double point isolated on $\mathbb{R}A$ with imaginary complex conjugate branches. A double real point with imaginary branches is not allowed in Theorem 1. Allow such points, but take into account their contribution to $w(\mathbb{R}A_+)$. Only one of the branches passing through it, belongs to $\mathbb{C}A_+$. Its intersection number with $\mathbb{R}^2$ is to be added to $w(\mathbb{R}A_+)$. Improved $w(\mathbb{R}A_+)$ is more invariant, and Theorem 1 holds true for it.

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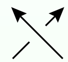
At a crossing an oriented link diagram looks either like this: 

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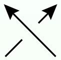
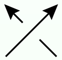
At a crossing an oriented link diagram looks either like this:  or like that: .

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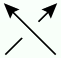
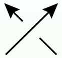
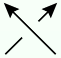
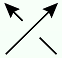
At a crossing an oriented link diagram looks either like this:  or like that: .

(Local) writhe: $w($$) = +1$, $w($$) - 1$.

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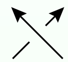
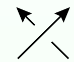
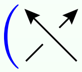
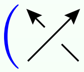
At a crossing an oriented link diagram looks either like this:  or like that: .

(Local) writhe: $w($$) = +1$, $w($$) - 1$.

Writhe of an oriented link diagram is the sum of local writhes over all crossings.

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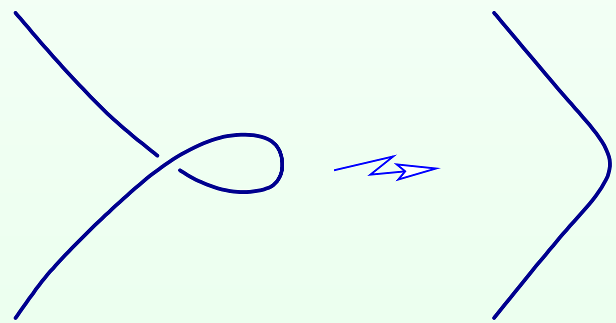
At a crossing an oriented link diagram looks either like this: $\nwarrow\!\!\!\!\nearrow$ or like that: $\nwarrow\!\!\!\!\nearrow$.

(Local) writhe: $w(\nwarrow\!\!\!\!\nearrow) = +1\,,\ w(\nwarrow\!\!\!\!\nearrow) - 1.$

Writhe of an oriented link diagram is the sum of local writhes over all crossings. It is not invariant: the first Reidemeister move



changes it by one.

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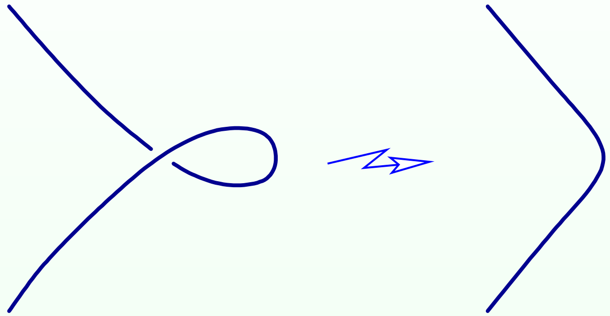
For an algebraic link the move



cannot happen.

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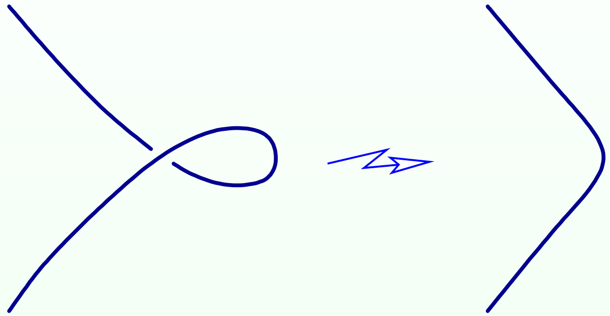
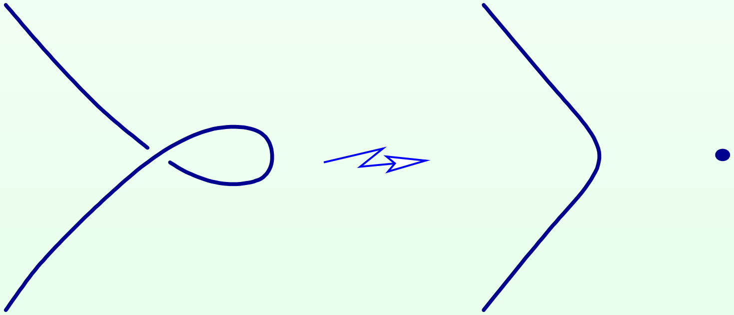
For an algebraic link the move 

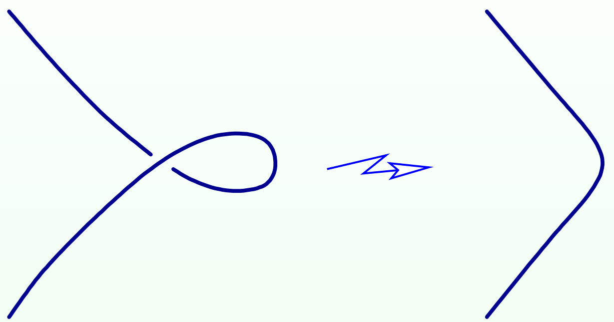
cannot happen.

The first real algebraic Reidemeister move looks like that:



encomplexed writhe

For an algebraic link the move



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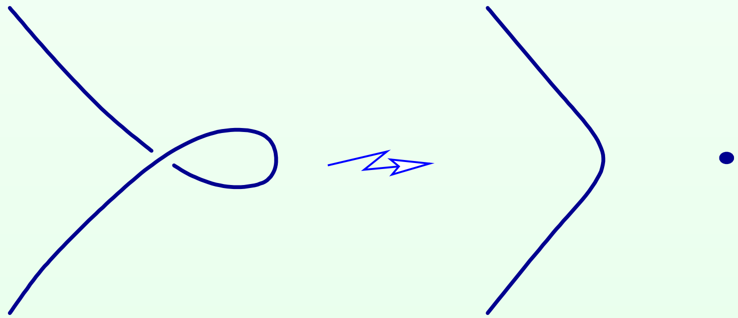

A crossing turns into a solitary real crossing of two complex conjugate imaginary branches.

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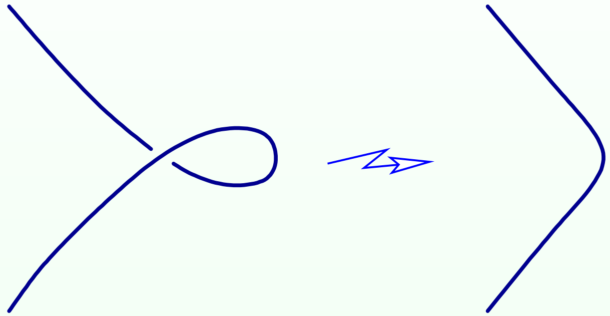
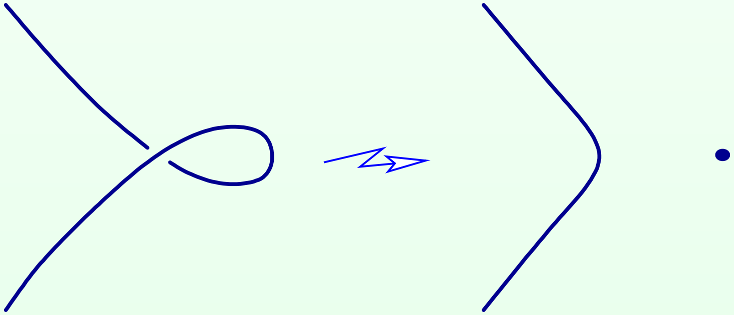
For an algebraic link the move  cannot happen.

The first real algebraic Reidemeister move looks like that:



A crossing turns into a solitary real crossing of two complex conjugate imaginary branches.

There is a writhe of a solitary crossing such that the total writhe does not change.

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An immersion $S^1 \looparrowright \mathbb{R}^2$ is *generic*, if it has neither triple point, nor a point of self-tangency.

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genericity of immersions

An immersion $S^1 \looparrowright \mathbb{R}^2$ is *generic*, if has neither triple point, nor a point of self-tangency. It has only ordinary double points of transversal self-intersection.

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A triple point of an immersion is *ordinary,* if the branches at the point are transversal to each other.

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A triple point of an immersion is *ordinary,* if the branches at the point are transversal to each other.

A self-tangency point of an immersion is *ordinary,* if the branches have distinct curvatures at the point.

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genericity of immersions

An immersion $S^1 \looparrowright \mathbb{R}^2$ is *generic*, if has neither triple point, nor a point of self-tangency. It has only ordinary double points of transversal self-intersection.

A triple point of an immersion is *ordinary,* if the branches at the point are transversal to each other.

A self-tangency point of an immersion is *ordinary,* if the branches have distinct curvatures at the point.

A self-tangency point of an immersion is called *direct,* if the velocity vectors are pointing the same direction;

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A self-tangency point of an immersion is called *direct,* if the velocity vectors are pointing the same direction; otherwise it is *inverse.*

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genericity of immersions

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A triple point of an immersion is *ordinary,* if the branches at the point are transversal to each other.

A self-tangency point of an immersion is *ordinary,* if the branches have distinct curvatures at the point.

A self-tangency point of an immersion is called *direct,* if the velocity vectors are pointing the same direction; otherwise it is *inverse.*

All non-generic immersions form a *discriminant hypersurface,* or just discriminant in the space of all immersions.

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The discriminant is stratified.

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The discriminant is stratified. There are 3 main open strata:

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The discriminant is stratified. There are 3 main strata:

- the set ST_+ of all immersions

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main strata of discriminant

The discriminant is stratified. There are 3 main strata:

- the set ST_+ of all immersions without triple points,

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The discriminant is stratified. There are 3 main strata:

● the set ST_+ of all immersions without triple points, with only one non-transversal double point,

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The discriminant is stratified. There are 3 main strata:

● the set ST_+ of all immersions without triple points, with only one non-transversal double point, and this is an ordinary direct self-tangency point.

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main strata of discriminant

The discriminant is stratified. There are 3 main strata:

● the set ST_+ of all immersions without triple points, with only one non-transversal double point, and this is an ordinary direct self-tangency point.

● the set ST_- of all immersions

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● the set ST_+ of all immersions without triple points, with only one non-transversal double point, and this is an ordinary direct self-tangency point.

● the set ST_- of all immersions without triple points,

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The discriminant is stratified. There are 3 main strata:

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● the set ST_- of all immersions without triple points, with only one non-transversal double point,

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The discriminant is stratified. There are 3 main strata:

● the set ST_+ of all immersions without triple points, with only one non-transversal double point, and this is an ordinary direct self-tangency point.

● the set ST_- of all immersions without triple points, with only one non-transversal double point, and this is an ordinary inverse self-tangency point.

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The discriminant is stratified. There are 3 main strata:

● the set ST_+ of all immersions without triple points, with only one non-transversal double point, and this is an ordinary direct self-tangency point.

● the set ST_- of all immersions without triple points, with only one non-transversal double point, and this is an ordinary inverse self-tangency point.

● the set TP of all immersions

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main strata of discriminant

The discriminant is stratified. There are 3 main strata:

● the set ST_+ of all immersions without triple points, with only one non-transversal double point, and this is an ordinary direct self-tangency point.

● the set ST_- of all immersions without triple points, with only one non-transversal double point, and this is an ordinary inverse self-tangency point.

● the set TP of all immersions which have only one triple point,

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main strata of discriminant

The discriminant is stratified. There are 3 main strata:

● the set ST_+ of all immersions without triple points, with only one non-transversal double point, and this is an ordinary direct self-tangency point.

● the set ST_- of all immersions without triple points, with only one non-transversal double point, and this is an ordinary inverse self-tangency point.

● the set TP of all immersions which have only one triple point, this point is ordinary,

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main strata of discriminant

The discriminant is stratified. There are 3 main strata:

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● the set ST_- of all immersions without triple points, with only one non-transversal double point, and this is an ordinary inverse self-tangency point.

● the set TP of all immersions which have only one triple point, this point is ordinary, besides this point, there are only ordinary double points.

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● the set ST_+ of all immersions without triple points, with only one non-transversal double point, and this is an ordinary direct self-tangency point.

● the set ST_- of all immersions without triple points, with only one non-transversal double point, and this is an ordinary inverse self-tangency point.

● the set TP of all immersions which have only one triple point, this point is ordinary, besides this point, there are only ordinary double points.

A generic path in the space of immersions (i.e. a generic regular homotopy)

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main strata of discriminant

The discriminant is stratified. There are 3 main strata:

● the set ST_+ of all immersions without triple points, with only one non-transversal double point, and this is an ordinary direct self-tangency point.

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● the set TP of all immersions which have only one triple point, this point is ordinary, besides this point, there are only ordinary double points.

A generic path in the space of immersions intersects the discriminant in a finite number of points,

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main strata of discriminant

The discriminant is stratified. There are 3 main strata:

● the set ST_+ of all immersions without triple points, with only one non-transversal double point, and this is an ordinary direct self-tangency point.

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● the set TP of all immersions which have only one triple point, this point is ordinary, besides this point, there are only ordinary double points.

A generic path in the space of immersions intersects the discriminant in a finite number of points, these points belong to the main strata.

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Changes experienced by an immersion when it goes through one of the strata were called *perestrojkas* by Arnold.

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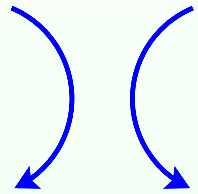

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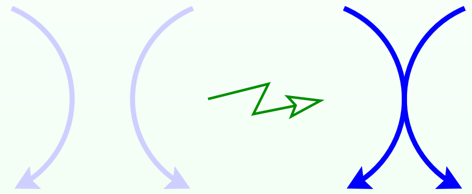

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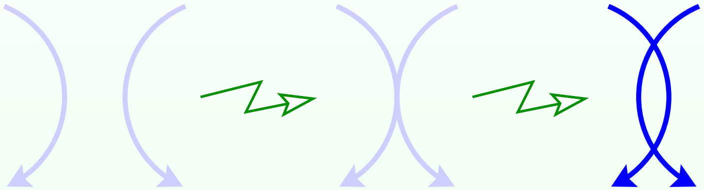

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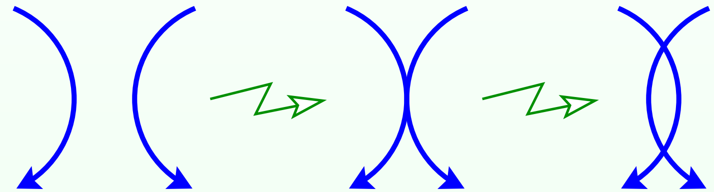
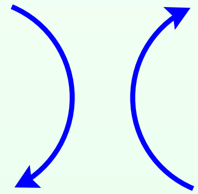


Direct self-tangency perestrojka. Passing through ST_+



Inverse self-tangency perestrojka. Passing through ST_-

perestrojkas

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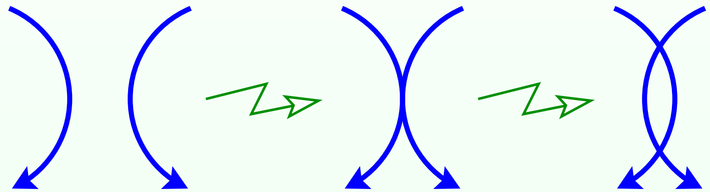
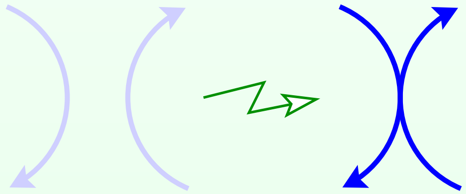


Direct self-tangency perestrojka. Passing through ST_+



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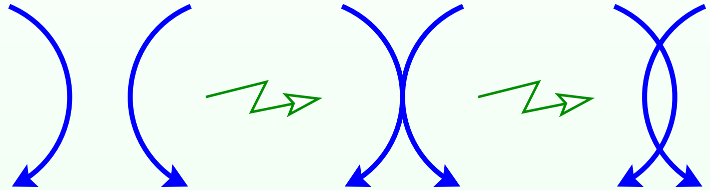

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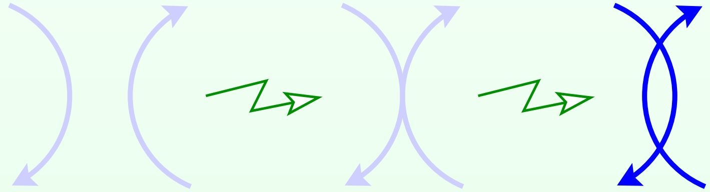

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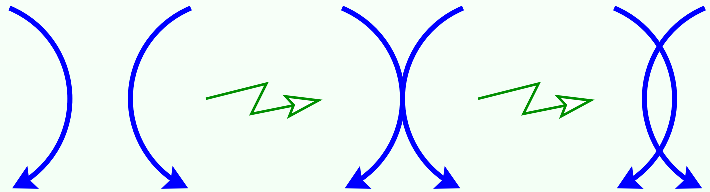

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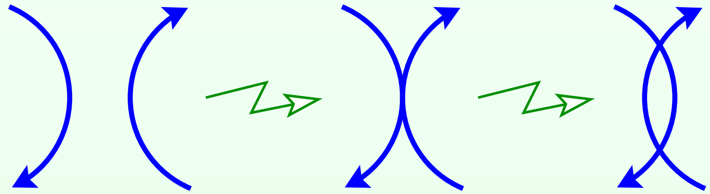
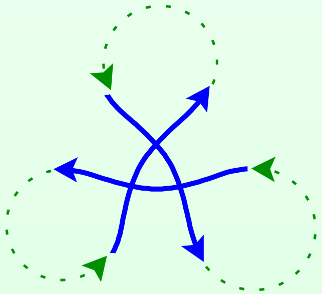


Inverse self-tangency perestrojka. Passing through ST_-



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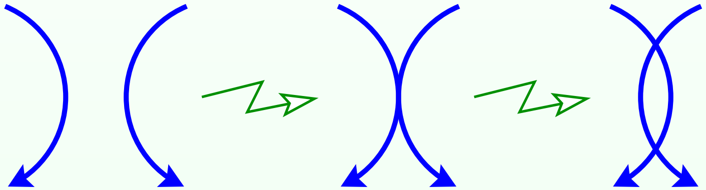

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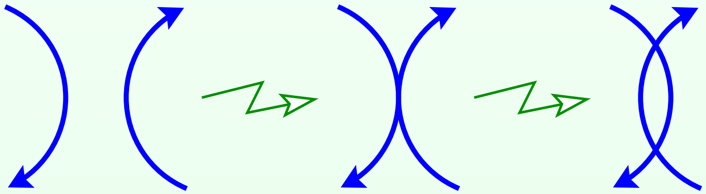
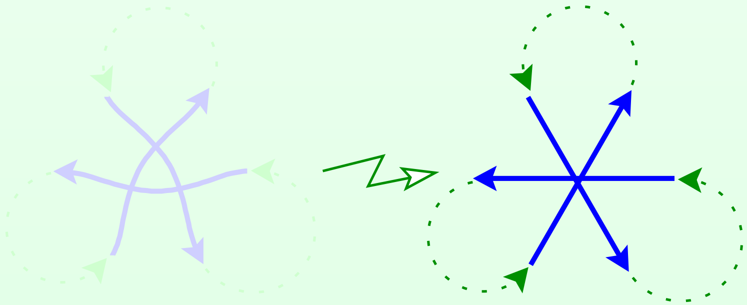


Inverse self-tangency perestrojka. Passing through ST_-



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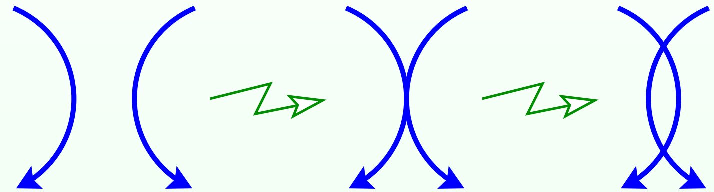
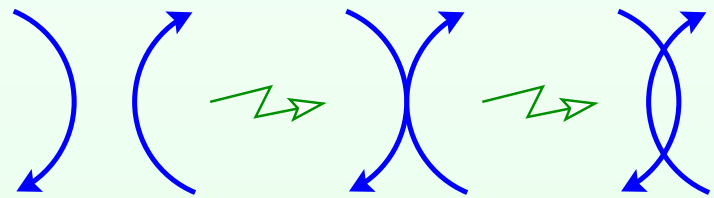
Changes experienced by an immersion when it goes through one of the strata were called *perestrojkas* by Arnold.



Direct self-tangency perestrojka. Passing through ST_+



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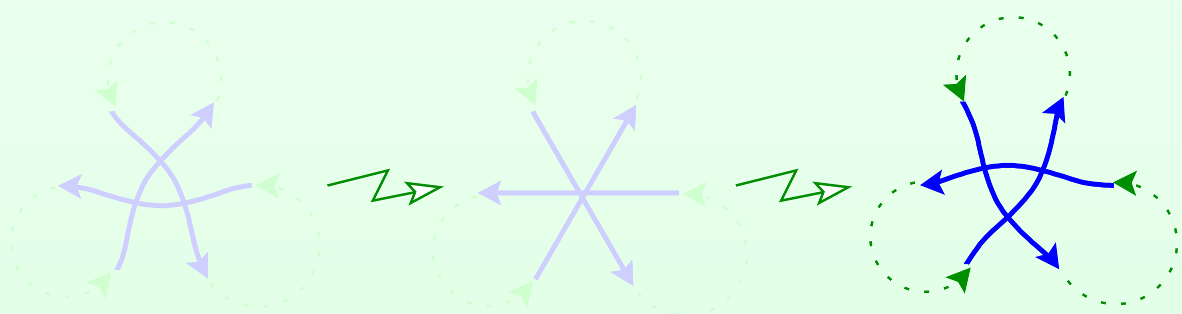

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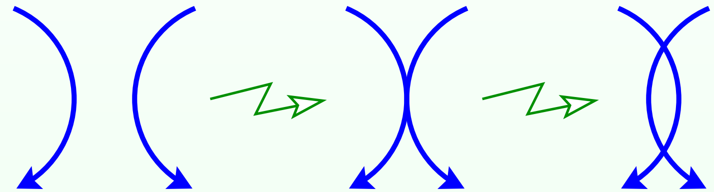
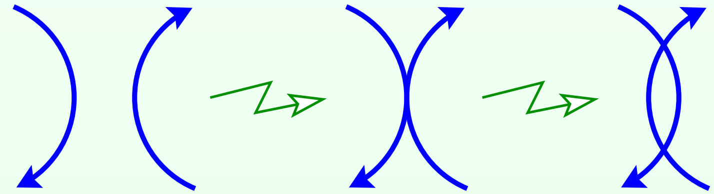
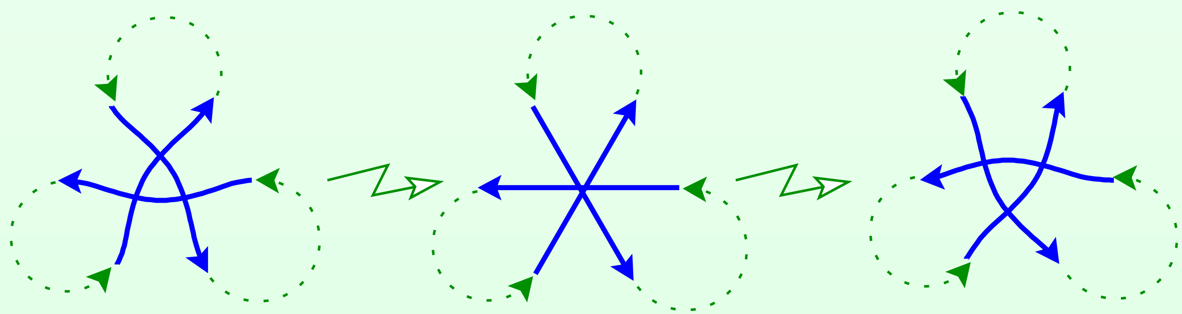


Direct self-tangency perestrojka. Passing through ST_+



Inverse self-tangency perestrojka. Passing through ST_-



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Arnold's invariants

For generic $C : S^1 \looparrowright \mathbb{R}^2$, Arnold introduced numerical characteristics $J^+(C)$, $J^-(C)$ and $St(C)$ defined by the following properties:

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For generic $C : S^1 \looparrowright \mathbb{R}^2$, Arnold introduced numerical characteristics $J^+(C)$, $J^-(C)$ and $St(C)$ defined by the following properties:

● invariance under regular homotopy in the class of generic immersions.

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● the following increments under perestojkas:

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Arnold's invariants

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● the following increments under perestojkas:

perestrojka	J_+	J_-	St
direct self-tangency	$+2$	0	0

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- invariance under regular homotopy in the class of generic immersions.
- the following increments under perestojkas:

perestrojka	J_+	J_-	St
direct self-tangency	$+2$	0	0
inverse self-tangency	0	-2	0

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● invariance under regular homotopy in the class of generic immersions.

● the following increments under perestojkas:

perestrojka	J_+	J_-	St
direct self-tangency	$+2$	0	0
inverse self-tangency	0	-2	0
triple point	0	0	$+1$

Arnold's invariants

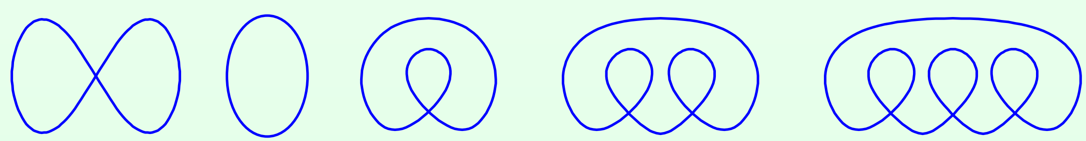
For generic $C : S^1 \looparrowright \mathbb{R}^2$, Arnold introduced numerical characteristics $J^+(C)$, $J^-(C)$ and $St(C)$ defined by the following properties:

● invariance under regular homotopy in the class of generic immersions.

● the following increments under perestojkas:

perestrojka	J_+	J_-	St
direct self-tangency	$+2$	0	0
inverse self-tangency	0	-2	0
triple point	0	0	$+1$

● For curves



K_0 K_1 K_2 K_3 K_4 $\ldots$

the invariants take the following values:

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For generic $C : S^1 \looparrowright \mathbb{R}^2$, Arnold introduced numerical characteristics $J^+(C)$, $J^-(C)$ and $St(C)$ defined by the following properties:

- invariance under regular homotopy in the class of generic immersions.
- the following increments under perestojkas:

perestrojka	J_+	J_-	St
direct self-tangency	$+2$	0	0
inverse self-tangency	0	-2	0
triple point	0	0	$+1$

$$J^+(K_0) = 0, \quad J^+(K_{i+1}) = -2i \quad (i = 0, 1, \dots);$$
$$J^-(K_0) = -1, \quad J^-(K_{i+1}) = -3i \quad (i = 0, 1, \dots);$$
$$St(K_0) = 0, \quad St(K_{i+1}) = i \quad (i = 0, 1, \dots).$$

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Consider irreducible real curves of degree d

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Consider irreducible real plane projective curves of degree d

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Consider irreducible real plane projective curves of degree d, genus g

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Consider irreducible real plane projective curves of degree d, genus g and type I

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Consider irreducible real plane projective curves of degree d, genus g and type I , equipped with complex orientations.

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Consider irreducible real plane projective curves of degree d, genus g and type I , equipped with complex orientations.

A generic curve A of this kind has only non-degenerate double singular points

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Consider irreducible real plane projective curves of degree d, genus g and type I , equipped with complex orientations.

A generic curve A of this kind has only non-degenerate double singular points , they can be of the following 4 types:

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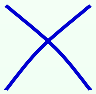
Consider irreducible real plane projective curves of degree d, genus g and type I , equipped with complex orientations.

A generic curve A of this kind has only non-degenerate double singular points , they can be of the following 4 types:

- real double points with two real branches ,

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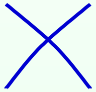
● real double points with two real branches  ,

● solitary real double point with two imaginary conjugate branches,

isolated point in $\mathbb{R}A$, local normal form $x^2 + y^2 = 0$.

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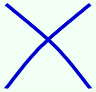
● real double points with two real branches ,

● solitary real double point with two imaginary conjugate branches,

At a solitary ordinary double point, the choice of $\mathbb{C}A_+$ determines a local orientation of $\mathbb{R}P^2$

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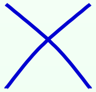
Consider irreducible real plane projective curves of degree d, genus g and type I , equipped with complex orientations.

A generic curve A of this kind has only non-degenerate double singular points , they can be of the following 4 types:

- real double points with two real branches  ,
- solitary real double point with two imaginary conjugate branches,

At a solitary ordinary double point, the choice of $\mathbb{C}A_+$ determines a local orientation of $\mathbb{R}P^2$
such that $\mathbb{R}P^2$ equipped with this local orientation intersects $\mathbb{C}A_+$ at this point with intersection number $+1$.

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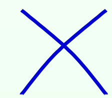
- real double points with two real branches ,
- solitary real double point with two imaginary conjugate branches,

At a solitary ordinary double point, the choice of $\mathbb{C}A_+$ determines a local orientation of $\mathbb{R}P^2$.

Another way to get the local orientation:
perturb the curve keeping type I and converting the solitary point into an oval.

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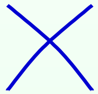
Consider irreducible real plane projective curves of degree d, genus g and type I , equipped with complex orientations.

A generic curve A of this kind has only non-degenerate double singular points , they can be of the following 4 types:

- real double points with two real branches ,
- solitary real double point with two imaginary conjugate branches,

At a solitary ordinary double point, the choice of $\mathbb{C}A_+$ determines a local orientation of $\mathbb{R}P^2$.

Another way to get the local orientation:
perturb the curve keeping type I and converting the solitary point into an oval. The complex orientation of this oval gives the local orientation of $\mathbb{R}P^2$.

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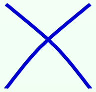
Consider irreducible real plane projective curves of degree d, genus g and type I , equipped with complex orientations.

A generic curve A of this kind has only non-degenerate double singular points , they can be of the following 4 types:

- real double points with two real branches  ,
- solitary real double point with two imaginary conjugate branches,
- imaginary double point of self-intersection of $\mathbb{C}A_+$,

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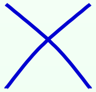
- real double points with two real branches ,
- solitary real double point with two imaginary conjugate branches,
- imaginary double point of self-intersection of $\mathbb{C}A_+$,
- imaginary intersection point of $\mathbb{C}A_+$ and $\mathbb{C}A_-$.

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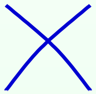
Consider irreducible real plane projective curves of degree d, genus g and type I , equipped with complex orientations.

A generic curve A of this kind has only non-degenerate double singular points , they can be of the following 4 types:

- real double points with two real branches ,
- solitary real double point with two imaginary conjugate branches,
- imaginary double point of self-intersection of $\mathbb{C}A_+$,
- imaginary intersection point of $\mathbb{C}A_+$ and $\mathbb{C}A_-$. Denote the number of the latter points by σ.

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Generic $\mathbb{R}A$ experiences perestrojkas considered above plus the following three new ones.

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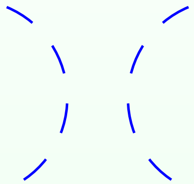

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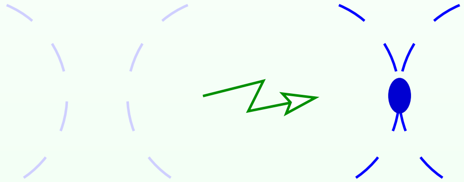

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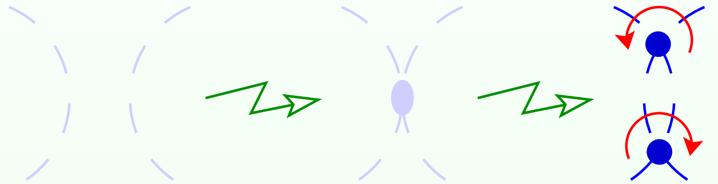

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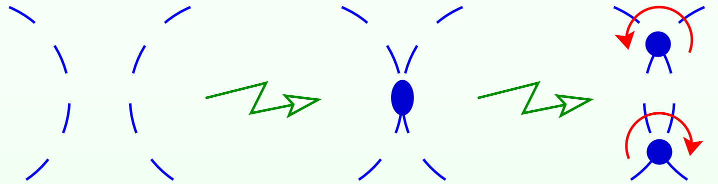

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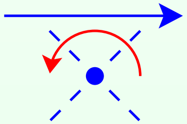

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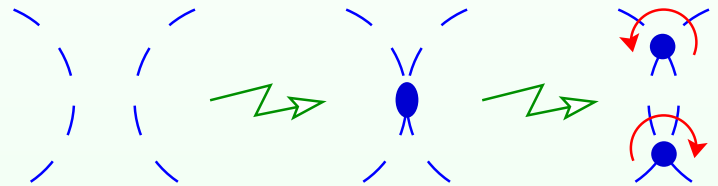

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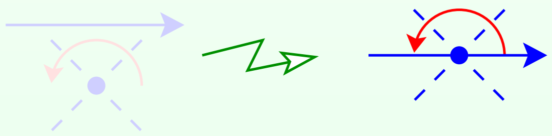

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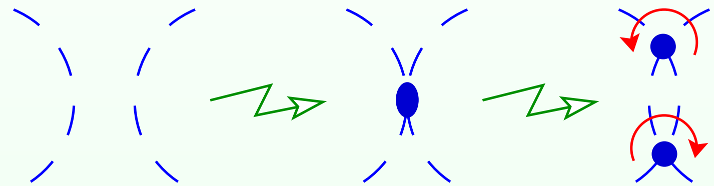
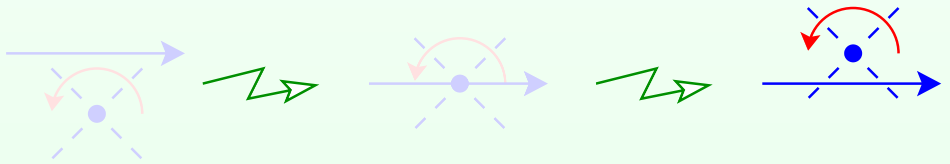


Solitary self-tangency perestrojka.



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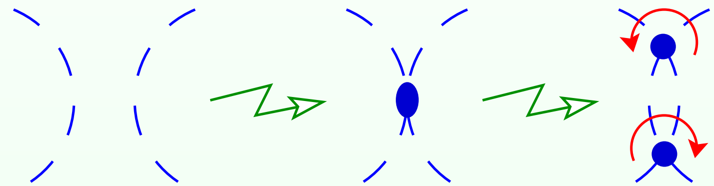

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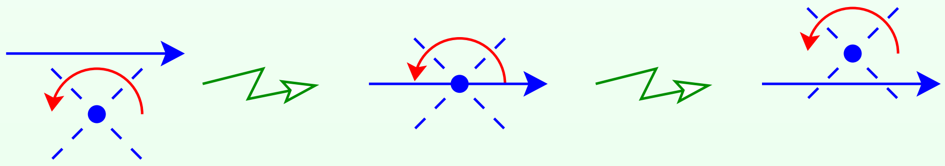

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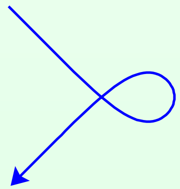

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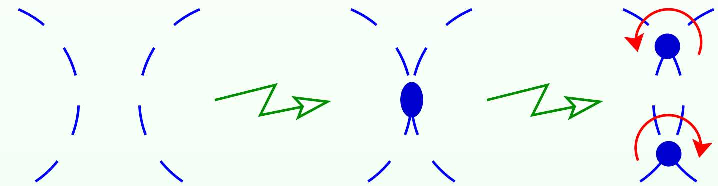
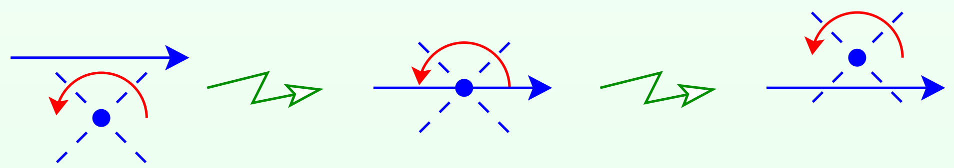


Solitary self-tangency perestrojka.



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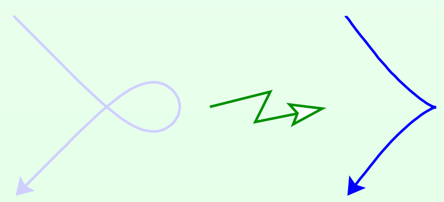

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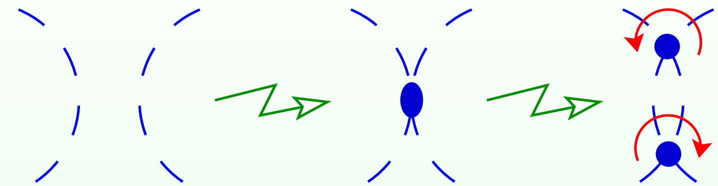
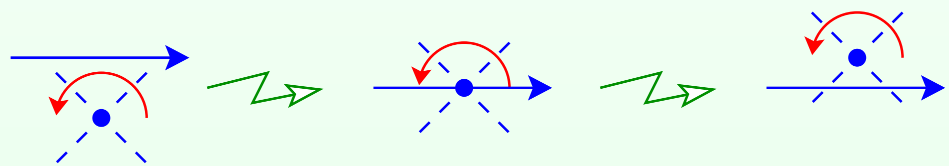
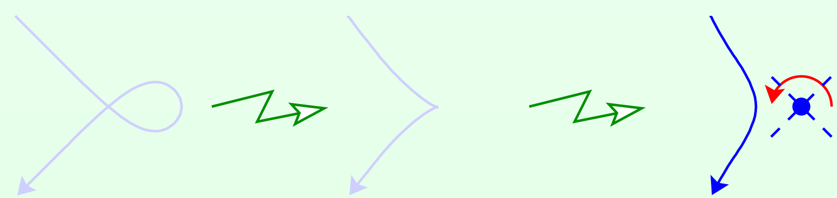


Solitary self-tangency perestrojka.



Triple point perestrojka with two imaginary branches.



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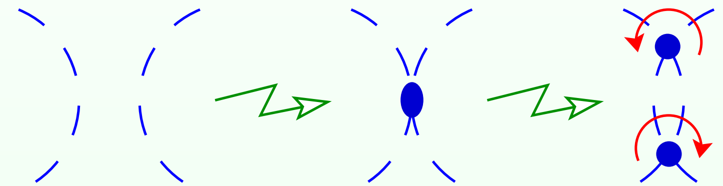

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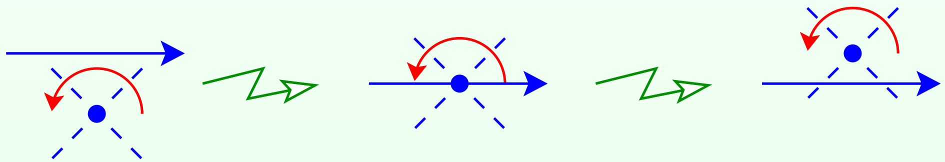

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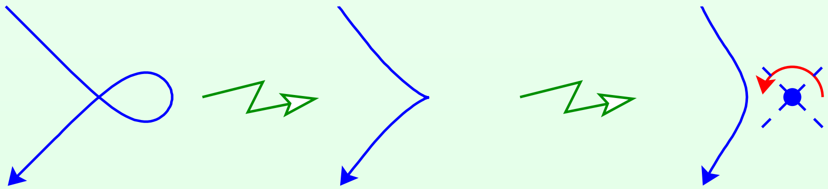

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Smoothing of curve

Smoothen a generic curve of type I according to the complex orientation: $A \mapsto \widetilde{A}$

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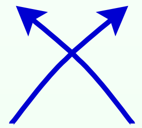

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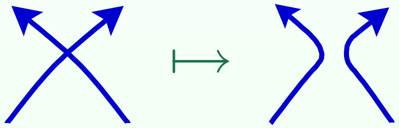

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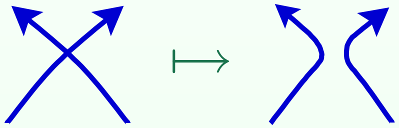

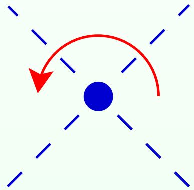

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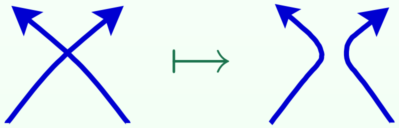
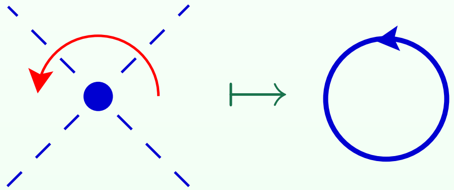
Smoothing of curve

Smoothen a generic curve of type I according to the complex orientation: $A \mapsto \widetilde{A}$





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For oriented closed curve $C \subset \mathbb{R}P^2$ and $x \in \mathbb{R}P^2 \setminus C$,

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For oriented closed curve $C \subset \mathbb{R}P^2$ and $x \in \mathbb{R}P^2 \setminus C$, define non-negative integer or half-integer $ind_C(x)$:

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For oriented closed curve $C \subset \mathbb{R}P^2$ and $x \in \mathbb{R}P^2 \smallsetminus C$, define non-negative integer or half-integer $ind_C(x)$: C realizes $2 \cdot ind_C(x)$-fold generator of $H_1(\mathbb{R}P^2 \smallsetminus \{x\}) = \mathbb{Z}$.

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Examples:

1. $ind_{\mathbb{R}P^1}(x) = \frac{1}{2}$

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Examples:

1. $ind_{\mathbb{R}P^1}(x) = \frac{1}{2}$
2. If C is a circle $x_1^2 + x_2^2 = x_0^2$ and x is a point in the disk bounded by C, then $ind_C(x) = 1$ independently on orientation of C.

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For oriented closed curve $C \subset \mathbb{R}P^2$ and $x \in \mathbb{R}P^2 \smallsetminus C$, define non-negative integer or half-integer $ind_C(x)$: C realizes $2 \cdot ind_C(x)$-fold generator of $H_1(\mathbb{R}P^2 \smallsetminus \{x\}) = \mathbb{Z}$.

Examples:

1. $ind_{\mathbb{R}P^1}(x) = \frac{1}{2}$
2. If C is a circle $x_1^2 + x_2^2 = x_0^2$ and x is a point in the disk bounded by C, then $ind_C(x) = 1$.
3. If C consists of two concentric circles, and x is their common center, then $ind_C(x)$ is either 0 or 2.

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Complex orientation formula

Let A be generic real plane projective algebraic curve of degree d and type I.

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Complex orientation formula

Let A be generic real plane projective algebraic curve of degree d and type I.

Then

$$\frac{d^2}{4} = \sigma + \int_{\mathbb{R}P^2 \smallsetminus \widetilde{\mathbb{R}A}} (ind_{\widetilde{\mathbb{R}A}}(x))^2 \, d\chi(x)$$

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here σ is the number of imaginary double points of A, where $\mathbb{C}A_+$ and $\mathbb{C}A_-$ meet,

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here σ is the number of imaginary double points of A, where $\mathbb{C}A_+$ and $\mathbb{C}A_-$ meet,
and the integral is against the Euler characteristic.

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Integral $\int f(x) \, d\chi(x)$ is defined for f which is a finite linear combination of characteristic functions,

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$f = \sum_{i=1}^{r} \lambda_i \mathbb{1}_{S_i}$, by formula

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$$\int f(x)\, d\chi(x) = \sum_{i=1}^{r} \lambda_i \chi(S_i).$$

intersection of complex halves

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Denote by σ the number of imaginary intersection points of $\mathbb{C}A_+$ and $\mathbb{C}A_-$ and study its behavior under perestrojkas.

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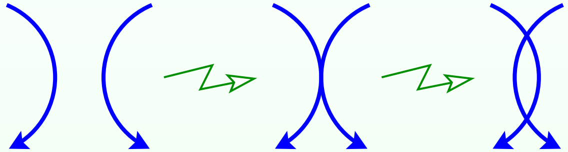

, σ does not change.

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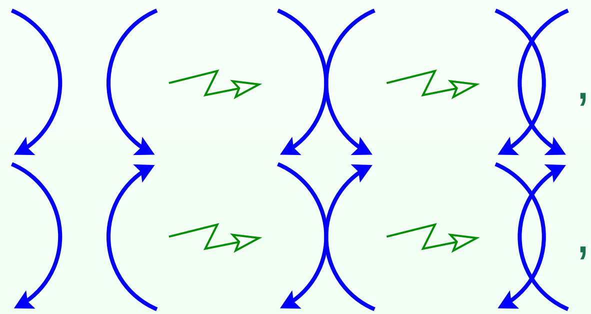

σ does not change.

σ decreases by 2.

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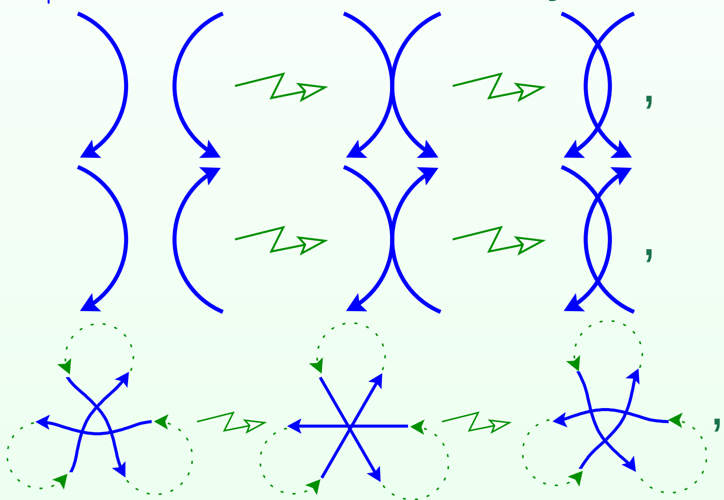

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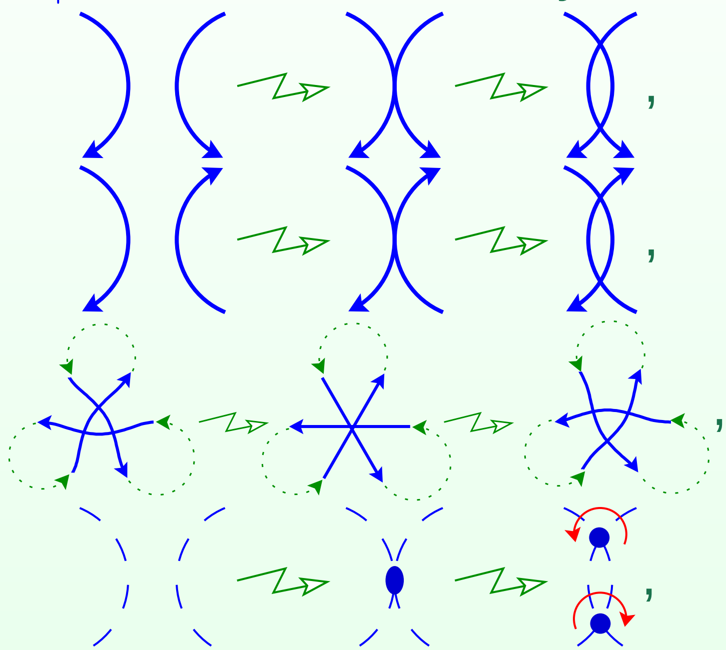

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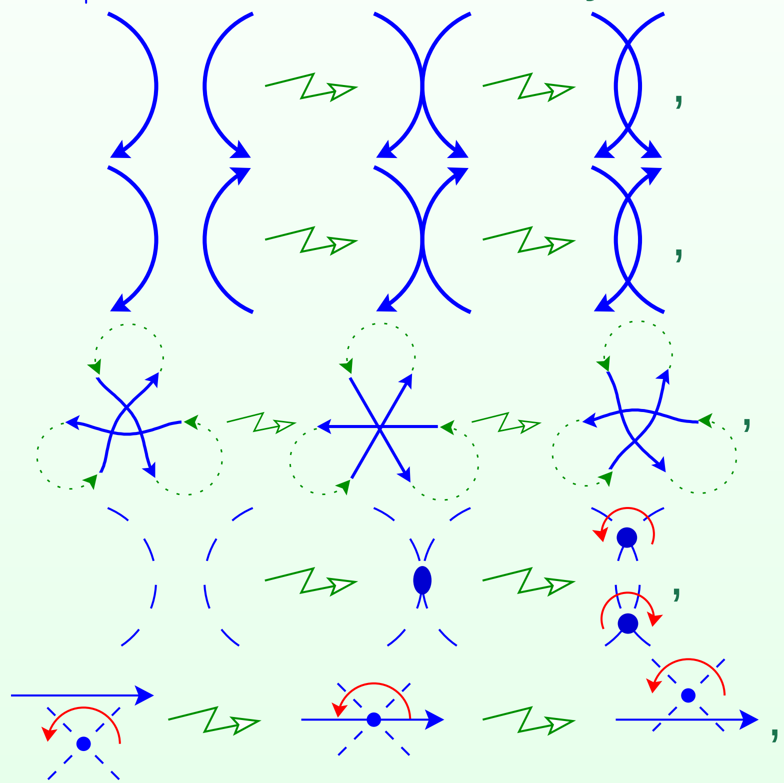

σ does not change.

σ decreases by 2.

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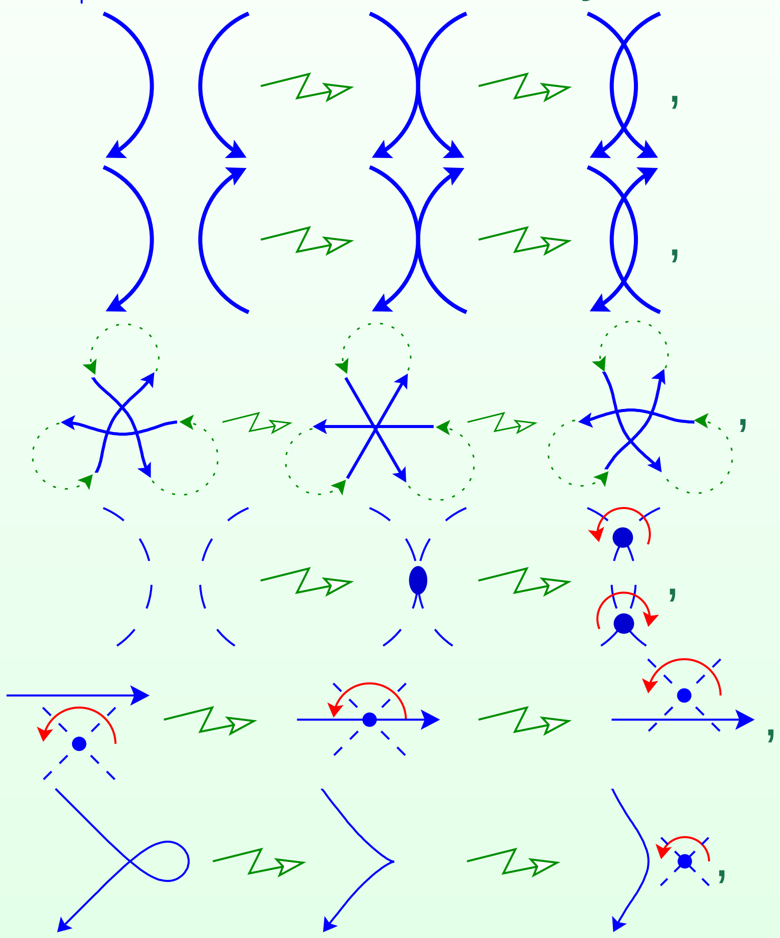
intersection of complex halves

Denote by σ the number of imaginary intersection points of $\mathbb{C}A_+$ and $\mathbb{C}A_-$ and study its behavior under perestrojkas.



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encomplexing J_-

Notice that σ behaves in the same way as J_- under direct and inverse self-tangency and triple point perestrojkas with only real branches involved.

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encomplexing J_-

Notice that σ behaves in the same way as J_- under direct and inverse self-tangency and triple point perestrojkas with only real branches involved.

Thus, σ can be considered as an encomplexed J_-.

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Notice that σ behaves in the same way as J_- under direct and inverse self-tangency and triple point perestrojkas with only real branches involved.

Thus, σ can be considered as an encomplexed J_-.

Complex orientation formula can be rewritten as a formula for σ:

$$\sigma = \frac{d^2}{4} - \int_{\mathbb{R}P^2 \smallsetminus \widetilde{\mathbb{R}A}} (ind_{\widetilde{\mathbb{R}A}}(x))^2 \, d\chi(x).$$

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Integral $-\int_{\mathbb{R}P^2 \smallsetminus \widetilde{\mathbb{R}A}} (ind_{\widetilde{\mathbb{R}A}}(x))^2 \, d\chi(x)$ has the same behavior under direct and inverse self-tangency and triple point perestrojkas as σ and J_-.

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This suggests to compare $J_-(C)$ with

$$-\int_{\mathbb{R}^2 \setminus \tilde{C}} (ind_{\tilde{C}}(x))^2 \, d\chi(x)$$

for a generic immersed circle C.

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Integral $-\int_{\mathbb{R}P^2\smallsetminus\widetilde{\mathbb{R}A}}(ind_{\widetilde{\mathbb{R}A}}(x))^2\,d\chi(x)$ has the same behavior under direct and inverse self-tangency and triple point perestrojkas as σ and J_-.

This suggests to compare $J_-(C)$ with

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for a generic immersed circle C.

Theorem. *For any generic immersed circle* C

$$J_-(C)=1-\int_{\mathbb{R}^2\smallsetminus\widetilde{C}}(ind_{\widetilde{C}}(x))^2\,d\chi(x).$$

J_+

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Corollary. *For any generic immersed circle* C *with* n *double points*

$$J_+(C) = 1 + n - \int_{\mathbb{R}^2 \smallsetminus \widetilde{C}} (ind_{\widetilde{C}}(x))^2 \, d\chi(x).$$

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The beginning of the story

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The beginning of the story , or the end of it?

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