Stony Brook University
Mathematics Department
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Topology, Geometry I
MAT 530
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Homework 3

1. Let A, B be open sets in a topological space X. Prove that if $A \cup B$ and $A \cap B$ are connected then A and B are connected.

2. Is the assumption that A and B are open necessary in the preceding problem?

3. Let A, B be open sets in a topological space X. Prove that if $A \cup B$ and $A \cap B$ are path connected then A and B are path connected.

4. Let X be a Hausdorff topological space and $f : X \to X$ be a continuous map. Prove that the set $\{x \in X \mid f(x) = x\}$ is closed in X.

5. Prove that the set of connected components of an open set on the plane $\mathbb{R}^2$ is countable.