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Topology, Geometry I
MAT 530
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Homework 1

1. Let S be a set, X be a set of all subsets of S. For $A \in X$ (that is $A \subset S$), denote by $U(A)$ the set of all subsets of S containing A. In formula:

$$U(A) = \{B \subset S \mid A \subset B\}.$$

Prove that $\{U(A) \mid A \subset S\}$ is a base of topology on X.

Let us call a collection $\mathcal{F}$ of closed sets in a topological space a ***c-base***, if any closed set is the intersection of some family of sets from $\mathcal{F}$.

2. Find a metric space in which the collection of closed balls is not a c-base.

3. What is the minimal number of points in such metric space?

4. Describe a c-base (not coinciding with the set of all closed sets) of an arbitrary metric space.

5. Let X be a topological space, A a subspace of X. Assume that for every continuous map $f : A \to Y$ to every topological space Y each map $g : X \to Y$ such that $f = g|_A$ is continuous. Reformulate this assumption entirely in terms of the topological structure of X, without mentioning any other topological structure.