

Homework 7

1. Prove that continuous maps $f, g : \mathbb{R} \rightarrow X$ are homotopic iff $f(0)$ and $g(0)$ belong to the same path-connected component of X .
2. Let $f, g : X \rightarrow S^2$ be continuous maps. Prove that if $f(x) \neq g(x)$ for each $x \in X$, then g is homotopic to the map $X \rightarrow S^2 : x \mapsto -f(x)$.
3. Let X be a topological space consisting of 3 points, a , b and c . Let points a and c be closed and point b everywhere dense. Is X path-connected? Find the fundamental group(s) of X .