Homework 7

1. Prove that continuous maps \( f, g : \mathbb{R} \to X \) are homotopic iff \( f(0) \) and \( g(0) \) belong to the same path-connected component of \( X \).

2. Let \( f, g : X \to S^2 \) be continuous maps. Prove that if \( f(x) \neq g(x) \) for each \( x \in X \), then \( g \) is homotopic to the map \( X \to S^2 : x \mapsto -f(x) \).

3. Let \( X \) be a topological space consisting of 3 points, \( a, b \) and \( c \). Let points \( a \) and \( c \) be closed and point \( b \) everywhere dense. Is \( X \) path-connected? Find the fundamental group(s) of \( X \).