Stony Brook University
Mathematics Department
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Topology, Geometry I
MAT 530
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Homework 6

1. Prove that the graph of any continuous function $[0,1] \to \mathbb{R}$ is closed, connected, path-connected, Hausdorff and compact.

2. Let X and Y be topological spaces, and Y be compact and Hausdorff. Prove that $f : X \to Y$ is continuous if and only if its graph $\Gamma_f = \{(x, y) \in X \times Y \mid y = f(x)\}$ is closed.

3. Let Z be a compact topological space, X and Y be Hausdorff topological spaces. Prove that if there exist continuous maps $f : Z \to X$ and $g : Z \to Y$ such that for any $a \in X$ and $b \in Y$ there exists a unique $c \in Z$ such that $f(c) = a$ and $g(c) = b$, then Z is homeomorphic to $X \times Y$.