Stony Brook University
Mathematics Department
Oleg Viro

Topology, Geometry I
MAT 530
September 29, 2012

Homework 5

1. Let X be a Hausdorff topological space and $f : X \to X$ be a continuous map. Prove that the set $\{x \in X \mid f(x) = x\}$ is closed in X.

2. Prove that the set of connected components of an open set on the plane $\mathbb{R}^2$ is countable.

3. Let K be the set of real numbers that are sums of series of the form $\sum_{k=1}^{\infty} \frac{a_k}{3^k}$ with $a_k \in \{0, 2\}$. Prove that K is uncountable, closed on the line $\mathbb{R}$ and totally disconnected.

4. Prove that if a topological space X contains compact sets A and B such that $A \cap B$ is not compact, then X is not Hausdorff.

5. Find a topological space X and compact sets $A, B \subset X$ such that $A \cap B$ is not compact.