Stony Brook University
Mathematics Department
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Topology, Geometry I
MAT 530
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Homework 4

1. Prove that a cover Γ of a topological space X is fundamental if each element of Γ is closed in X and any point $a \in X$ has a neighborhood U which has non-empty intersection only with a finite number of elements of Γ.

2. Let A and B be connected sets and A contain a boundary point of B. Prove that $A \cup B$ is connected.

3. Let A, B be open sets in a topological space X. Prove that if $A \cup B$ and $A \cap B$ are connected then A and B are connected.

4. Is the assumption that A and B are open necessary in the preceding problem?

5. Let A, B be open sets in a topological space X. Prove that if $A \cup B$ and $A \cap B$ are path connected then A and B are path connected.