

Homework 3

1. Revisit all the definitions in sections 2, 3, 4, 5, 6, 10 and 11. Be prepared for reproducing any of the definitions in writing.
2. Prove that a map $f : X \rightarrow Y$ is continuous if and only if $\text{Cl } f(A) \supset f(\text{Cl } A)$ for any set $A \subset X$.
3. Prove that the metric topology on a metric space (X, ρ) is the smallest topological structure for which the functions $X \rightarrow \mathbb{R} : x \mapsto \rho(a, x)$ are continuous for all $a \in X$.
4. In an arbitrary metric space, what is relation between the boundary of an open ball $B_r(c)$ and sphere $S_r(c)$ of the same radius and center? Do they coincide? Or one of them is contained in the other? Give complete proofs for your answer.