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Topology, Geometry I
MAT 530
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Homework 1

Let S be a set, X be a set of all subsets of S. For $A \in X$ (that is $A \subset S$), denote by $U(A)$ the set of all subsets of S containing A. In formula:

$$U(A) = \{B \subset S \mid A \subset B\}.$$

1. Prove that $\{U(A) \mid A \subset S\}$ is a base of topology on X.

2. Describe the closure of a point $A \in X$ with respect to this topology.

Let us call a collection $\mathcal{F}$ of closed sets in a topological space a ***c-base***, if any closed set is the intersection of some family of sets from $\mathcal{F}$.

3. Find a metric space in which the collection of closed balls is not a c-base.

4. What is the minimal number of points in such metric space?

5. Describe a c-base (not coinciding with the set of all closed sets) of an arbitrary metric space.