Algebra for Teachers Name _____
Homework 2
Due 2/17 Score _____

Present a complete solution for each problem. Answers alone will give no credit.

1. Let G be a multiplicative group. Prove that $(ab)^{-1} = b^{-1}a^{-1}$ for all $a, b \in G$.

2. Let G be a multiplicative group. Prove that if $a^2 = 1$ for all $a \in G$, then G is commutative.

3. Let $G = \mathbb{R} \setminus \{1\}$. Define operation $*$ on G as follows: $a * b = a + b - ab$. Prove that $(G, *)$ is a commutative group. (You have to prove that G is closed with respect to $*$ and check that three axioms of a group are satisfied. And check commutativity, too.)

4. Let $3\mathbb{Z} = \{3n \mid n \in \mathbb{Z}\}$. Which of the standard properties have the addition and multiplication on $3\mathbb{Z}$? Is this an additive group? Is this a multiplicative group?

5. Calculate $(211)_3 \cdot (12)_3$ in the positional system with base 3. Check your calculations using decimal system, that is find the decimal presentations for the two given numbers, multiply them and convert the product to a base 3 system.